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Sir R. S. BALL, LL.D., F.R.S., President, in the Chair.

John Franklin-Adams, Wimbledon ;
Rev. Sherrard Beaumont Burnaby, M.A., The Vicarage, Hampstead, N.W. ;
John Sisson Slater, M.A., LL.D., Seafield House, Lytham, and 1 Garden Court, Temple, E.C. ; and
Rev. John Spence, 27 Walpole Street, Chelsea, S.W.,

were balloted for and duly elected Fellows of the Society.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended :—

Professor Henry C. Lord, Director of the Emerson McMillin Observatory, Ohio State University, Columbus, Ohio, U.S.A. (proposed by George E. Hale).

Seventy-one presents were announced as having been received since the last meeting, including, amongst others—

H. Jacoby, Reduction of Stellar Photographs, &c., presented by Columbia College Observatory ; Meudon, Observatoire d'Astronomie Physique, Annales, tome i., presented by the Observatory ; S. Newcomb, Action of the Planets on the Moon, and Secular Variations of the Orbits of the Four Inner Planets,

presented by the office of the *American Ephemeris*; W. J. S. Lockyer, Resultate aus Beobachtungen des veränderlichen Sternes *η Aquilæ*; W. Prinz, Esquisses Sélénologiques, presented by the authors; two MS. volumes containing original entries of Capt. Foster's pendulum observations, 1828–31, presented by the Hydrographer.

On some Original Unpublished Observations of the Comet of 1652.

By E. B. Knobel.

In a copy of the second edition of Bayer's *Uranometria* (Ulm, 1639), with which is bound the *Cœlum Stellatum Christianum* of Julius Schiller (Augsburg, 1627), which I have recently acquired, there are recorded several observations of the comet of 1652, and as they appear to be for the most part the original observations of the former owner of the work, they are perhaps worth saving from the obscurity in which they have been concealed since that period.

The comet of 1652 was very carefully observed by Hevelius, who devotes the whole of the first book of his "Cometographia" to an exhaustive discussion of his observations, and it was from these materials that the elements of the comet's orbit were subsequently computed by Halley, and published in his well-known "Synopsis," which appeared in the *Philosophical Transactions* for 1705.* Besides his own very important series, Hevelius mentions some observations made by Wendelinus at Tournai, Malvaticæus at Bologna, Christianus at Giessen in Hesse, and by an Englishman at Aleppo, but it is clear that neither of these can be the author of the observations I have discovered. It is probable, however, that the author is indicated in an inscription on the title-page of Bayer, in the same handwriting as that of the observations. This inscription gives an epitome of the title-page of the "Coelum Stellatum," and concludes with the sentence: "A. 1649 pro isto Coelo Stellato petebat D(ominus) Henri(cus) Milius ? nuper."

This author had evidently the open maps before him, on which he each night marked down the position of the comet by alignment with the stars, and he wrote on the maps particulars of such alignment with notes and remarks, as well as some observations made elsewhere by others.

Hevelius observed the comet from 1652 December 20 to 1653 January 8. Our unknown author gives the positions of the comet from December 19 to January 3, and there is a note which

* In his "Constantes de l'Astronomie" (*Annales de l'Observatoire de Bruxelles*) Houzeau gives Halley's longitude of perihelion erroneously as 38° 10′ 40″; it should be 28° 10′ 40″.

indicates that the comet was observed as late as 1653, January 20. These dates are new style.*

From several references in the notes to observations at Cologne, and also to some made at a monastery in Westphalia, I infer that the author may probably have lived at Cologne, where I found the work ; indeed, as a matter of fact, the book belonged to the library of a Burgomaster at Cologne when I bought it.

The positions of the comet marked on the maps being those derived solely from alignment with the stars, they are of course of the same epoch as *Bayer's Uranometria*, which is that of *Tycho's Catalogue*, A.D 1601. I have taken out the longitudes and latitudes from the maps, and reduced the longitudes to 1653, and compared the results with those observations of Hevelius nearest in time, and as these latter were made, as is well known, at Danzig, I have corrected the times given by my unknown author (assumed as for Cologne) to the meridian of Danzig.

The following are the positions and notes exactly as given in the manuscript, on the maps :—

	Longitude.	Latitude.	Notes.
	° ′	° ′	
1652 Dec. 19	68 40	36 20 S.	19 Dec. inter pedes *Orionis*.
„ 20	66 30	30 20 S.	20 Dec. Bononiæ in Italia h. $3\frac{1}{2}$ post ☉ occasum.
„ 20	66 20	29 30 S.	20 Dec. contra genu occidentale *Orionis*.
„ 21	62 40	18 25 S.	21 Dec. 1652 h. $7\frac{1}{4}$ post merid. Monasterij Westphaliæ efficiebat Triang. æquilaterū cū duabus scūt *Orionis* maxime distantibus secundum Henr. Moderton. (?) qui addit 3 fere gr. ad polū ab æquatore.
„ 21	...	...	21 Dec. contra scutum *Orionis* Colon. Studiosi.
„ 21	62 30	17 30 S.	Circa hunc locū visus cometes 21 Dec. a Studiosis Colon. Vespere diei S. Thomæ.
„ 22	60 10	9 30 S.	22 Dec. h.
„ 22	60 20	9 30 S.	22 Dec. 1652 Hora 8 sero.
„ 26	53 20	13 25 N.	26 Dec. 1652 Sero h. 9. Colon.
„ 26	53 15	13 20 N.	26 Dec. 1652 in Latit. 37 fere visus Cometa sero h. 9.
„ 27	52 35	15 10 N.	27 Dec. h. 5 mane prope occasū et ideo cū parallaxi quam Dec. 27 eodē cetexit vesp̄tina obseruatis.
„ 27	...	...	Mane 27 Dec. hora 5 erat inter duas rectas duobus a Pleiadibus ad Lucidissimam in pectore Persei et ab ijsdem ad Lucidissimam capitis Medusæ ducta verò linea per utramqué stellam pedis.

* There is a possibility of uncertainty about dates in Germany at this period, for though new style was introduced into that country so early as 1584, it was not generally adopted till 1699 November.

	Longitude.	Latitude	Notes.
1652 Dec. 27	52 15	16 30 N.	27 Dec. sero h. 10 exacte obseruaui crucē notatam in Lat. 40 f.
„ 27	52 0	16 30 N.	27 Dec. mihi 25 ab oculo Tauri.
„ 28	52 30	17 30 N.	28 Dec. 1652 Mane h. 5 Coloniæ apparebat Cometa.
„ 28	51 20	19 0 N.	28 Dec. sero h. 9 Lat. B. 42½.
„ 29	51 20	21 40 N.	29 Dec. Vesp. h. 8 deficiens Lat. B. 45.
„ 30	...	...	30 Dec. h. 2 post mediam noctē stetit in hac linea inter oculū Medusæū et lucidam in genu Persei ductâ sic ut attingeret pene oculum Medusæ, ex occidente imo mane eodem h. 6 iam subingressus eundem oculū ita obnubilabat eum, ut is esset obscurior quam stella p̄xima supra genu.
„ 30	50 25	23 30 N.	30 Dec. h. 6 sero Lat. B. 47 f.
1653 Jan. 3	48 20	28 35 N.	3 Jan. mane h. 4 Latit. 51½ fere.
„ 20	36 20?	43 35 N.?	Hoc triangulū æquilaterum 6½ gr. Iabi longiore efficiebat Monasterij Westphaliæ 20 die Januarij ad vesperā hora 8.

Positions reduced to 1653, *and the times corrected to the meridian of Danzig, compared with Hevelius* :—

	Auctor in Bayer.			*Hevelius.*		
Date.	Time.	Long.	Lat.	Time.	Long.	Lat.
	h m	° ′	° ′	h m	° ′	° ′
1652 Dec. 19	?	69 24	36 20 S.	...	...	...
„ 20	8 21 P.M.	67 14	30 20 S.	7 0 P.M.	68 24	30 49 S.
„ 20	?	67 4	29 30 S.	...	...	...
„ 21	8 2 P.M.	63 24	18 25 S.	...	...	...
„ 22	?	60 54	9 30 S.	...	...	...
„ 22	8 47 P.M.	61 4	9 30 S.	...	...	...
„ 26	9 47 P.M.	54 4	13 25 N.	9 0 P.M.	54 0	13 24 N.
„ 26	9 47 P.M.	53 59	13 20 N.			
„ 27	5 47 A.M.	53 19	15 10 N.	2 6 A.M.	54 17	13 53 N.*
„ 27	10 47 P.M.	52 59	16 30 N.	11 44 P.M.	53 23	17 5 N.
„ 27	?	52 44	16 30 N.	...	...	...
„ 28	5 47 A.M.	53 14	17 30 N.	0 13 A.M.	53 14	17 6 N.
„ 28	9 47 P.M.	52 4	19 0 N.	...	...	...
„ 29	8 47 P.M.	52 4	21 40 N.	5 0 A.M.	52 21	20 33 N.
„ 30	6 47 P.M.	51 9	23 30 N.	6 10 P.M.	51 28	23 23 N.
1653 Jan. 3	4 47 A.M.	49 4	28 35 N.	6 47 P.M.	50 10	29 2 N.
„ 20	8 47 P.M.	37 4	43 35 N.			

* Hevelius's Latitude is clearly in error and that in the MS. is more correct.

The comet was not observed by Hevelius on December 19, 21, and 22.

With respect to the above observations, that recorded as made at Bologna on December 20 was probably by Cornelius Malvaticæus mentioned by Hevelius. The comet is indicated as on a line with β *Eridani* and 27 *Orionis*. On December 21, at 8.2 p.m. Danzig time, the comet formed an equilateral triangle with π^3 and π^5 *Orionis*. On December 22 it is shown between μ and ρ *Tauri*, and nearly on a line with them. On December 27, probably at 10.47 p.m. Danzig time, the author gives the angular distance of the comet from α *Tauri* as 25°. Hevelius measured the same angle at 5^h 38^m Danzig time the same evening and made it 24° $16\frac{1}{2}'$. On this date, at 10.47 p.m. Danzig time, the author shows the comet exactly on a line joining o and ρ *Persei*, and also on a line joining 39 *Arietis* and ϵ *Persei*, these lines being nearly at right angles, and thus form the cross referred to under this date.

On December 28, at 9.47 p.m. Danzig time, the comet is shown between ϵ *Persei* and δ *Trianguli* and exactly on the line joining them. On December 29, at 8.47 p.m. Danzig time, it is on the line joining ξ and β *Persei*, and also on the line joining δ and ω *Persei*. On December 30, at 2 hours "post mediam noctem," 2.47 a.m. Danzig time, the comet was on the line joining ϵ and β *Persei*; and on January 3 the comet made a right angle with α and β *Persei*, and was situated slightly south of a line joining α *Persei* and γ *Andromedæ*.

With respect to the observation dated January 20 at 8 p.m., "Hoc triangulum æquilaterum $6\frac{1}{2}$ gr. labi longiore efficiebat Monasterii Westphaliæ 20 die Januarii ad vesperam hora 8," the point marked on the map is about $6\frac{1}{2}°$ from each of the two stars in the Sword of Perseus, marked *v* and *g* in Bayer, = Fl. 1 and Fl. 4, which are also about $6\frac{1}{2}°$ from each other. But there must necessarily be a doubt whether the comet was really observed in this position. From December 23 to January 7 the position of the comet had altered in longitude only 9°, whereas in latitude it had moved 35°. On this latter date it was about 2° south of τ *Persei*, where it was last seen by Hevelius. From January 4 to January 7 the apparent motion of the comet was slow, in a path passing between γ and τ *Persei*; and it is extremely difficult to understand how it could be in the position assigned on January 20. If it be true, it is an important observation; but there is a suspicion of a mistake, seeing that the assigned place on January 20 is very close to two stars θ and μ *Cassiopeiæ*, which are within 52′ of longitude and 18′ of latitude of each other, and in hazy weather might thus have led the observer to believe that he saw the comet. The Moon was not above the horizon at the time.

Hevelius observed the comet from December 20 to January 8, but in his Table of Comets he states that it lasted till 1653 January 10; and he further makes a statement respecting the distance of the comet "vero 12 Januarii." His remarks on its

motion, as translated by Sherburne (*Manilius*, p. 210), are :— "Its motion was constantly retrograde, from South to North, by the Hare, Foot of *Orion*, *Taurus*, to the *Pleïades* ; and from thence as far as *Perseus* ; not directly but obliquely from East South East to North North West, in a great circle inclined to the Ecliptick, and Æquator : At first by its diurnal motion compleating 11° 16′, at last 30′, it ran through in the time of its duration 65° 51′ passing beyond and beside the course of the Annual Orb."

Hevelius declares positively that the comet disappeared after January 10. His words are : "Quamobrem serio sum persuasus ultimo hâc die esse visum ; et postmodum à nemine observatum esse : quemadmodum et mihi, die 11 Januarii, nusquam apparuit." There is a further marginal note : "Nullus, præter autorem, cometam ad 11 usque Januarii observavit."

The observations of December 21, 26, 27, 28, 29, 30, and January 3, which state that the "latitude" of the comet was 3°, 37°, 40°, $42\frac{1}{2}$°, 45°, 47°, and $51\frac{1}{2}$° respectively, must not be taken as the latitude from the ecliptic ; but they are what Hevelius terms "Distantia ab intersectione Æquatoris"—that is to say, the angular distance of the comet from the equator in its apparent orbit considered as a great circle.*

On the Straightness of Spider Lines.

By H. H. Turner, M.A., B.Sc., Savilian Professor.

1. The gist of the following note is that the horizontal wire of the Greenwich Transit Circle is sensibly straight. I was led to examine this point from hearing that a curvature or "sag" of the horizontal wire had been detected elsewhere. If such a curvature existed it would specially affect the R—D observations, and might form a considerable part of this curious phenomenon.

2. The method adopted was to take from the section of observed zenith-distances in the volumes of Greenwich Observations for 1892 and 1893 all those cases in which a star had been observed in zenith-distance at the vertical wires 1, 2, 6, 7. The readings of the micrometer at wires 1 and 7 were subtracted

* In the copy of Bayer under consideration, the author of the observations has designated the group of stars 33, 35, 39, and 41 *Arietis* as "Nubes *Tauri* vel Halitus *Arietis*." These stars formed the constellation "Musca," or "Apes," of Jacobus Bartchius, the son-in law of Kepler, and I am unaware of any other designation for them than his. The explanation of "Nubes *Tauri*" is that the stars are situated on the confines of the cloud which conceals the body of the Bull, as shown in the old drawings of the constellations in Aratus and Hyginus, and in Bayer. "Halitus *Arietis*" is explained by the face of the Ram being turned in the direction of these stars, and thus the animal's breath would cover them.

from those at wires 2 and 6. If the path of the star across the field were a straight line, and there were any "sag" of the wire, the result (2+6)—(1+7) would be positive. The inclination of the wire to the horizontal would be eliminated.

3. The path of a star is, however, curved, and the correction for curvature is given in the Greenwich Observations as

$$+0^{r}\cdot00327 \times \cot \text{N.P.D.} \times (\text{wire}-4)^{2},$$

the unit 1^{r} being a revolution of the telescope micrometer, $=37''\cdot05$, so that $^{r}\cdot00327 = 0''\cdot121$ (it is convenient to keep our figures in revolutions of the micrometer). Thus the correction to the quantity tabulated (2+6)—(1+7) is

$$+\{(2^2+2^2)-(3^2+3^2)\} \times {}^{r}\cdot00327 \times \cot \text{N.P.D.}$$

or

$$-{}^{r}\cdot0327 \times \cot \text{N.P.D.},$$

which was accordingly applied to the results.

4. In the following table the results are given, arranged according to N.P.D. The approximate mean N.P.D. of the group is given in the *first* column, each group including all the stars within 10° of N.P.D. In the *second* column is given the curvature correction corresponding to this mean N.P.D., to give an idea of its magnitude. The observations were, however, each corrected separately, and not by use of this mean correction. In the *third* column is given the flexure factor sin Z.D. It seems probable that if there were any sag in the wire, it would be greater at large zenith-distances, according to some law like that of flexure.

N.P.D.	Correction for Curvature.	Factor for Flexure.	Readings at wires (2+6)—(1+7); corrected for curvature. 1892₁.	No. Obs.	1892₂.	No. Obs.	1893.	No. Obs.	Mean.	No. Obs.
°	r		r		r		r		r	
45	−0·033	0·10	−·015	8	−·016	6	−·002	7	−·011	21
55	−0·022	0·28	−·003	14	−·001	6	−·011	13	−·006	33
65	−0·015	0·44	−·002	37	−·002	10	+·001	18	−·001	65
75	−0·009	0·59	−·004	20	−·004	16	+·004	17	−·001	53
85	−0·003	0·72	+·016	9	+·005	12	+·001	11	+·007	32
95	+0·003	0·83	−·002	12	+·007	14	+·001	9	+·002	35
105	+0·009	0·91	+·001	10	+·011	8	+·006	22	+·006	40
115	+0·015	0·97	+·003	6	−·007	19	−·006	13	−·005	38
123	+0·022	1·00	+·048	2	−·004	4	−·018	9	−·005	15

In columns 4, 6, 8, are given the mean results of three series of groups, corresponding to the dates 1892 January 1–June 30, 1892 July 1–December 31, 1893 January 1–December 31: the number of stars in each group being shown in columns 5, 7, and 9. Columns 10 and 11 give the weighted means (according to number of observations) and weights for all the stars.

5. There appears to be no evidence of curvature in the wire. The simple mean of column 10 is $-^{r}\cdot0016$, and the weighted mean $-^{r}\cdot0008$; equivalent to $-0''\cdot06$ and $-0''\cdot03$ respectively ; and indicating an *upward* curvature if anything. If we equate the results of column 10 to k sin Z.D., and solve by least squares, we get

$$k = -^{r}\cdot0003 = -''\cdot01.$$

6. The negative value for 45° is noteworthy, especially as the few available observations on the north side of the zenith also give negative results, as follows—

-45°	$-^{r}\cdot018$	-36°	$-^{r}\cdot051$	$+25^{\circ}$	$-^{r}\cdot082$	$+28^{\circ}$	$-^{r}\cdot009$
-42	$-\cdot042$	-35	$+\cdot007$	$+25$	$-\cdot032$	$+35$	$-\cdot011$

But there are not enough observations to settle anything definitely about this side of the zenith. There is one way in which such negative results for stars above the pole may be spurious, viz. if the observer inadvertently transposes the readings of the micrometer at wires 1 and 2, or at 6 and 7. Such errors would give negative results in the above investigation.

7. It occurred to me as possible that the law assumed for curvature of the star's path might not be sufficiently accurate near the pole ; but a very little examination showed that it is accurate enough except for close polar stars, which are not usually observed far from the meridian at Greenwich. I take this opportunity of pointing out another case where the use of rectangular coordinates simplifies the problem very much.

8. The accurate formula for correcting for curvature is given by Chauvenet (vol. ii. p. 291) as

$$-\frac{\sin^2\frac{1}{2}(\tau-m)}{\sin 1''}\cdot\sin 2\delta' - \frac{2\sin^4\frac{1}{2}(\tau-m)}{\sin 1''}\sin 2\delta'\sin^2\delta', \text{ \&c.}$$

where τ is the east hour angle of the star, and c, m and n are the usual instrumental constants. It will be noted that c and n do not appear in this expression. If we use rectangular coordinates x and y along the horizontal and vertical wires respectively ; then, since the path of the star is an ellipse (the projection of a small circle on a plane) its equation is

$$y = ax^2 + by^2.$$

To find a and b, perhaps the simplest plan is to take three rectangular axes S O, S P, and a perpendicular to both ; where O is the centre of the celestial sphere, O P the polar axis, S the star, and S P a tangent to the sphere at S. The coordinates of O are (0, 0 1) if S P be taken as axis of y and S O of z ; and the coordinates of P are (0, tan p, 0) ; so that the direction cosines of O P are proportional to (0, tan p, − 1). Let (x, y) be any point

Q of the star's path traced on the tangent plane. The direction cosines of O Q are proportional to $(x, y, -1)$; and thus, since this line is inclined to O P at angle p we have

$$\cos p = \frac{y \tan p + 1}{\sqrt{(1+\tan^2 p)(1+x^2+y^2)}},$$

or

$$1 + x^2 + y^2 = 1 + 2y \tan p + y^2 \tan^2 p$$

or

$$y = \tfrac{1}{2} \cot p \,.\, x^2 + \cot 2p \,.\, y^2.$$

When p is small, this may be written

$$y = \frac{x^2}{2p} + \frac{x^4}{8p^3} = x\left[\left(\frac{x}{2p}\right) + \left(\frac{x}{2p}\right)^3\right]$$

all the quantities being expressed in circular measure.

The distances of wires 1 and 7 from the meridian at Greenwich are about 11′; so that for a star 1° from the pole $\frac{x}{2p} = \cdot 09$, and $\left(\frac{x}{2p}\right)^3 = \cdot 0007$; thus the second term produces an effect of $11 \times 60 \times \cdot 0007 = 0''\cdot 5$, which is quite appreciable. But at 2° from the pole this becomes $0''\cdot 06$; so that it is negligible except for close polar stars.

9. We have in the above neglected the instrumental constants c, m, n, and the inclination of the wire. The effect of the constants c, m, n is to shift the apparent origin of coordinates to the right by the quantity

$$(c + m \sin p + n \cos p) \equiv c' \text{ say},$$

assuming that c, m, n are expressed in seconds of arc, and thus we have

$$y = \tfrac{1}{2}(x - c')^2 \cot p + y^2 \cot 2p.$$

But c' is very small. The chief additional term is $-c'x \cot p$, which is eliminated in symmetrical transits (at wires 1 and 7 or 2 and 6 say), and does not therefore affect the above investigation concerning $(2+6)-(1+7)$ for instance. The constant term $\frac{1}{2}c'^2 \cot p$ is very small indeed, and in any case does not affect the above investigation.

On the Apparent Disc and on the Shadow of an Ellipsoid.

By A. Marth.

If k denotes the equatorial radius and $e = \sin \epsilon_0$ the eccentricity of a planet's ellipsoidal surface, so that the polar radius is $k \cos \epsilon_0$, the outline of the planet's disc, as seen from a point in planetocentric latitude B above the plane of the planet's equator and at the distance Δ, will be an ellipse, the semi-axes a and b of which are

$$a = \frac{k}{\Delta}\omega'' \qquad \omega'' = \frac{1}{\text{arc } 1''}$$

$$b = a\sqrt{1 - e^2 \cos^2 B}$$

$$= a \cos \epsilon,$$

If, further,

$$\left.\begin{aligned} \sin \epsilon &= \sin \epsilon_0 \cos B, \\ \cos \epsilon \sin \nu &= \sin \epsilon_0 \sin B \\ \cos \epsilon \cos \nu &= \cos \epsilon_0, \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (1)$$

the proportion $\cos \epsilon$ of the axes of the apparent disc is found from the proportion $\cos \epsilon_0$ of the axes of the ellipsoid by

$$\cos \epsilon = \cos \epsilon_0 \sec \nu, \text{ where } \tan \nu = \tan \epsilon_0 \sin B,$$

and $\cos \epsilon_0$ from $\cos \epsilon$ by

$$\cos \epsilon_0 = \cos \epsilon \cos \nu, \text{ where } \sin \nu = \tan \epsilon \tan B.$$

These simple formulæ for the outline of the apparent disc are only valid when the distance Δ in comparison with k is so great that terms of the second order $\frac{k^2}{\Delta} \ldots$ may be neglected. If that is not the case, strict formulæ must be substituted.

Let λ and β denote the planetocentric longitude and latitude, referred to the plane of the planet's equator, of a point on the ellipsoidal surface, $k\rho$ its linear distance from the centre of the ellipsoid, and L B Δ the corresponding coordinates of the point of view, so that L B and $k \cos \nu$ are the coordinates of the centre point of the apparent disc. Further, let $p - P = \pi$ denote the position angle of the surface point at the apparent centre of the disc, reckoned from the direction P of the polar axis, σ the planetocentric angular distance or the angle between $k\rho$ and Δ, Δ' the linear distance between the surface point and the point of view, and s the angle between Δ' and Δ. The values of $\pi\ \sigma\ s\ \Delta'$ are found by

$$\left.\begin{aligned} \sin \sigma \sin \pi &= \cos \beta \sin (L - \lambda) \\ \sin \sigma \cos \pi &= -\cos \beta \cos (L - \lambda) \sin B + \sin \beta \cos B \\ \cos \sigma &= \cos \beta \cos (L - \lambda) \cos B + \sin \beta \sin B \\ \Delta' \sin s &= k\rho \sin \sigma \\ \Delta' \cos s &= \Delta - k\rho \cos \sigma. \end{aligned}\right\} \quad \ldots \quad (2)$$

Instead of the latitudes β and B, it will be advantageous to introduce latitudes β' and B′ connected with them by

$$\tan \beta' = \tan \beta \sec \epsilon_0$$

and

$$\tan B' = \tan B \sec \epsilon_0.$$

These latitudes are the latitudes of the points, where lines passing through the surface points perpendicular to the equator cut the sphere circumscribed with radius k round the centre of the ellipsoid, or also of the points where lines perpendicular to the polar axis meet the sphere circumscribed with radius $k\cos\epsilon_0$, so that we have

$$\left.\begin{aligned} \rho \sin \beta &= \sin \beta' \cos\epsilon_0 \quad \text{and} \quad \cos \nu \sin B = \sin B' \cos \epsilon_0 \\ \rho \cos \beta &= \cos \beta' \qquad\qquad \cos \nu \cos B = \cos B' \\ &\qquad\qquad\qquad\qquad \sin \nu = \sin B' \sin \epsilon_0 \end{aligned}\right\} \quad . \quad . \quad . \quad (3)$$

and the equations (2) become

$$\left.\begin{aligned} \rho \sin \sigma \sin \pi &= \cos \beta' \sin (L-\lambda) \\ \rho \sin \sigma \cos \pi \,.\, \sec \epsilon &= -\cos \beta' \cos (L-\lambda) \sin B' + \sin \beta' \cos B' \\ \rho \cos \sigma \,.\, \cos \nu &= \cos \beta' \cos (L-\lambda) \cos B' + \sin \beta' \sin B' \,.\, \cos^2 \epsilon_0 \end{aligned}\right\} \quad (4)$$

In order that Δ' should be a tangent to the ellipsoid, the coordinates of the surface point, if for a few moments we introduce rectangular coordinates

$$\begin{aligned} \xi &= k\rho \cos \beta \cos \lambda & X &= \Delta \cos B \cos L \\ \eta &= k\rho \cos \beta \sin \lambda & Y &= \Delta \cos B \sin L \\ \zeta &= k\rho \sin \beta & Z &= \Delta \sin B, \end{aligned}$$

must satisfy the equation

$$1 = \frac{\xi X + \eta Y}{kk} + \frac{\zeta Z}{kk \cos^2 \epsilon_0},$$

or, returning to polar coordinates,

$$\frac{k}{\Delta} = \rho \cos \beta \cos (L-\lambda) \cos B + \rho \sin \beta \sin B \,.\, \sec^2 \epsilon_0,$$

or finally

$$\frac{k}{\Delta} \cos \nu = \cos \beta' \cos (L-\lambda) \cos B' + \sin \beta' \sin B' \quad . \quad . \quad . \quad (5)$$

From this equation and the equations (4), the special values of $\rho \sin \sigma'$ and $\rho \cos \sigma'$ for points of the rim are to be deduced, which, substituted in the last equation (2), furnish the corresponding values of s' and Δ'.

The sum of the squares of the first and second equation (4) and of (5) gives

$$\rho^2 \sin^2 \sigma \,(\sin^2 \pi + \cos^2 \pi \sec^2 \epsilon) + \frac{\lambda^2}{\Delta^2} \cos^2 \nu = 1$$

or

$$\rho^2 \sin^2 \sigma \,(1 + \tan^2 \epsilon \cos^2 \pi) = 1 - \frac{\lambda^2}{\Delta^2} \cos^2 \nu.$$

Therefore, putting

$$\left.\begin{aligned} \frac{k}{\Delta} \cos \nu &= \sin h \\ \tan \epsilon \cos \pi &= \tan \epsilon' \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (6)$$

we have

$$\rho \sin \sigma' = \cos h \cos \epsilon'.$$

The third equation (4) and (5) gives

$$\rho \cos \sigma' = \frac{k}{\Delta} - \sin \beta' \sin B' \,.\, \sin^2 \epsilon_0 \sec \nu$$

$$= \frac{k}{\Delta} - \tan^2 \epsilon_0 \,.\, \rho \sin \beta \sin B.$$

But

$$\sin \beta = \sin \sigma \cos \pi \cos B + \cos \sigma \sin B.$$

Hence

$$\rho \cos \sigma' \,(1 + \tan^2 \epsilon_0 \sin^2 B) = \frac{k}{\Delta} - \tan^2 \epsilon_0 \,.\, \rho \sin \sigma' \cos \pi \cos B \sin B,$$

or as

$$\rho \sin \sigma' = \cos h \cos \epsilon',$$

$$\rho \cos \sigma' = \sin h \cos \nu - \cos h \sin \nu \sin \epsilon'$$

Substituting these values of $\rho' \sin \sigma'$ and $\rho \cos \sigma'$ in the last equations (2) we derive

$$\left.\begin{aligned} \Delta' \sec h \sin s' &= k \cos \epsilon' \\ \Delta' \sec h \cos s' &= \Delta \cos h + k \sin \nu \sin \epsilon' \\ \text{or} \quad \frac{\Delta' \cos \nu}{\Delta \cos h} \sin s' &= \sin h \cos \epsilon' \\ \frac{\Delta' \cos \nu}{\Delta \cos h} \cos s' &= \cos h \cos \nu + \sin h \sin \nu \sin \epsilon' \\ \tan s' &= \frac{\tan h \sec \nu \cos \epsilon'}{1 + \tan h \tan \nu \sin \epsilon'} \end{aligned}\right\} \quad \ldots \quad (7)$$

or, if preferred, indirectly

$$\tan s' = \frac{k \cos \epsilon'}{\Delta \cos h} (1 - \tan s' \sin \nu \tan \epsilon')$$

For

$\pi = \pm 90°$ $\quad \epsilon'$ is 0 hence $\tan s' = \tan h \sec \nu$ or $= \frac{k}{\Delta} \sec h$

$\pi = \frac{0°}{180}$ $\quad \epsilon' = \pm \epsilon$ hence $\tan s' = \frac{\tan h \sec \nu \cos \epsilon}{1 \pm \tan h \tan \nu \sin \epsilon}$

$$= \frac{k \cos \epsilon}{\Delta \cos h} (1 \mp \tan s' \sin \nu \tan \epsilon)$$

As ν has the same sign as B, the radii s' of the disc on that side of the equator, where the point of view is, are accordingly rather shorter than those on the opposite side.

If the values of λ and β of the points of the rim are required, they may be found by means of the equations

$$\left.\begin{aligned} \sec h \cos \beta' \sin (L-\lambda) &= \cos \epsilon' \sin \pi \\ \sec h \cos \beta' \cos (L-\lambda) &= -\frac{\cos \epsilon'}{\cos \epsilon} \cos \pi \sin B' + \tan h \cos B' \\ \sec h \sin \beta' &= \frac{\cos \epsilon'}{\cos \epsilon} \cos \pi \cos B' + \tan h \sin B' \end{aligned}\right\} \quad (8)$$

The formulæ thus derived for the apparent disc of an ellipsoid may now be serviceable in the investigation of the shadow problem, the strict solution of which, when pursued in this way, will be found to offer no great obstacles, even if the light-supplying body is itself an ellipsoid of any eccentricity.

A few preliminary considerations may be useful for a clearer understanding of the question at issue. Let K be the radius of the (spherical) Sun, R the distance between Sun and planet, and let the point of view be in the prolongation of R and of the axis of the shadow. While the distance of the point from the centre of the planet is Δ, its distance from the Sun is $R+\Delta$, so that the Sun's apparent disc is a circle of semi-diameter H, where $\sin H = \frac{K}{R+\Delta}$, and upon this circle the dark disc of the planet is seen projected. Let now the point of view approach the planet or let Δ be lessened till the edges of the planet in position angles $\pm 90°$ touch the rim of the Sun. The Sun appears there in the shape of two cusps, the end points of which just touch, and the inner edges of which, being formed by the outline of the ellipsoid, are not symmetrical, except when B is 0, the cusp on the side of B being the broader one. By farther approaching the planet, the two discs cross, having tangents in common at the four crossing points. With Δ lessening the cusps diminish more and more, till the last rays of the Sun disappear, first on the side opposite to B, and finally at the end point of the disc's minor axis on the side of B. Now these groups of tangents, common to the two bodies at every point of the shadow axis within the indicated limits, form a continuous surface, the surface of the umbra, and it is required to determine the sections of this surface or the curves described by this surface on planes perpendicular to the shadow axis at any given points.

The angle s' between the tangent to the ellipsoid in position angle π and the shadow axis at a point at distance Δ, is found by

$$\tan s' = \frac{k \cos \epsilon'}{\Delta \cos h + k \sin \nu \sin \epsilon'},$$

the angle H between the tangent to the spherical Sun and the shadow axis by

$$\sin H = \frac{K}{R+\Delta}.$$

In order that the tangent should be common to both bodies, the distance Δ must be so determined that $s' = H$, or that

$$\sin s' = \frac{k \cos \epsilon' \cos s'}{\Delta \cos h + k \sin \nu \sin \epsilon'} = \frac{K}{R+\Delta},$$

and, therefore, that

$$\frac{k}{\Delta} = \frac{K \cos h - k \cos \epsilon' \cos s'}{R \cos \epsilon' \cos s' - K \sin \nu \sin \epsilon'}.$$

Hence we get the value of h by

$$\tan h = \frac{K \sec \epsilon' - k \sec h \cos s'}{R \sec \nu \cos s' - K \tan \nu \tan \epsilon'},$$

which, as it contains $\sec h$ and $\cos s'$, must be found indirectly if no conditions are introduced to make their differences from 1 insensible.

If r is the distance of a point of the tangent from the centre of the planet, and σ the angle between r and the shadow axis, the linear distance of the point from the axis will be

$$r \sin \sigma = (\Delta - r \cos \sigma) \tan s',$$

or as

$$\Delta \tan s' = \frac{k \cos \epsilon'}{\cos h + \sin h \tan \nu \sin \epsilon'},$$

$$r \sin \sigma = \frac{k - r \cos \sigma \,.\, \sin h \sec \nu}{\cos h \sec \epsilon' + \sin h \tan \nu \tan \epsilon'}.$$

The formulæ for finding the radii $r \sin \sigma$ of the curve formed by the umbra on a plane passing perpendicularly through the point $r \cos \sigma$ of the axis, are accordingly

$$\left.\begin{aligned} \tan \epsilon' &= \tan \epsilon \cos \pi \\ \tan h &= \frac{K \sec \epsilon' - k \sec h \cos s'}{R \sec \nu \cos s - K \tan \nu \tan \epsilon'} \\ \tan s' &= \frac{\tan h \sec \nu}{\sec \epsilon' + \tan h \tan \nu \tan \epsilon'} \\ r \sin \sigma &= \frac{k \sec h - r \cos \sigma \,.\, \tan h \sec \nu}{\sec \epsilon' + \tan h \tan \nu \tan \epsilon'} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (9)$$

Though the assumption that the illuminating body itself is an ellipsoid has at present no practical application, I will not fail to give also the formulæ adapted to that case.

Let K be the equatorial radius and $E = \sin \eta_0$ the eccentricity of the ellipsoidal Sun, B the latitude above the Sun's equator and $R + \Delta$ the distance of the point of view in the shadow axis, and

let the position angle of the polar axis of the planet, reckoned from the direction of the polar axis of the Sun, be P, so that the position angle of a tangent to the Sun corresponding to π is $\pi+\mathrm{P}$.

Making then

$$\tan\gamma=\tan\eta_0\cdot\sin B$$

$$\tan\eta=\tan\eta_0\cdot\cos B\sec\gamma$$

$$\tan\eta'=\tan\eta\cos(\pi+\mathrm{P})$$

and

$$\sin\mathrm{H}=\frac{\mathrm{K}\cos\gamma}{\mathrm{R}+\Delta},$$

the angle H′ between the tangent in position angle $\pi+\mathrm{P}$ and the shadow axis will be

$$\tan\mathrm{H}'=\frac{\mathrm{K}\cos\eta'}{(\mathrm{R}+\Delta)\cos\mathrm{H}+\mathrm{K}\sin\gamma\sin\eta'},$$

and this tangent will be common to both bodies, planet and Sun, if Δ is so determined that

$$\frac{k\cos\epsilon'}{\Delta\cos h+k\sin\nu\sin\epsilon'}=\frac{\mathrm{K}\cos\eta'}{(\mathrm{R}+\Delta)\cos\mathrm{H}+\mathrm{K}\sin\gamma\sin\eta'}.$$

Accordingly the value of

$$\frac{k\cos\nu}{\Delta\cos h}=\tan h$$

is found by

$$\tan h=\frac{\mathrm{K}\cos\eta'\sec\epsilon'-k\cos\mathrm{H}\sec h}{\mathrm{R}\cos\mathrm{H}+\mathrm{K}\cos\eta'(\sin\gamma\tan\eta'-\sin\nu\tan\epsilon')}\cdot\cos p$$

(the cos H involved being derived from

$$\sin\mathrm{H}=\frac{\mathrm{K}\cos\gamma}{\mathrm{R}+\dfrac{\sin h\sec\nu}{k}}\Bigg),$$

and finally the radius $r\sin\sigma$ in position angle π of the curve formed by the surface of the umbra on a plane passing perpendicularly through the point $r\cos\sigma$ of the shadow axis

$$r\sin\sigma=\frac{k\sec h-r\cos\sigma\cdot\tan h\sec\nu}{\sec\epsilon'+\tan h\sec\nu\cdot\sin\nu\tan\epsilon'}.$$

The application of these formulæ to fictitious cases leads to a variety of interesting curves, the consideration of which I must here forego.

In the cases of *Jupiter* and of *Saturn* the computation of the sections of their shadows becomes simplified. The "Ephemerides for Physical Observations of *Jupiter*" for 1891 and succeeding years contain the longitudes $\Lambda+180°$ and latitudes B of the Sun

(referred to the plane of *Jupiter's* equator and corrected for aberration), which, when used with the "Data for Computing the Positions of the Satellites of *Jupiter*," serve in computing the jovicentric coordinates

$$r \sin \sigma \sin \pi, \qquad r \sin \sigma \cos \pi, \qquad r \cos \sigma$$

of the satellites referred to the axis of the shadow cone as the z axis, as mentioned in *Monthly Notices*, vol. li. p. 508. The distance $r \sin \sigma$ of the satellite from the axis of the shadow cone must then be compared with the corresponding radius of the section of the shadow which passes through the point $r \cos \sigma$ of the z axis. The formulæ become ($-B$ being substituted for B)

$$\tan \nu = \tan \epsilon_0 \sin B$$
$$\tan \epsilon = \tan \epsilon_0 \cos B \sec \nu$$
$$\tan \epsilon' = \tan \epsilon \cos \pi$$
$$h = \frac{\mathrm{K} \sec \epsilon' - k}{\mathrm{R} - \mathrm{K} \sin \nu \tan \epsilon'}$$

(h being written for $\tan h \sec \nu$)

$$r \sin \sigma = \frac{k - h \,.\, r \cos \sigma}{\sec \epsilon' - h \,.\, \sin \nu \tan \epsilon'}.$$

In the case of the shadow of the Earth the formulæ become still simpler so far as the geometrical shadow is concerned.

Let A D be the right ascension and declination of the axis of the shadow, or of the prolongation of R, α δ the geocentric right ascension and declination of the Moon, π its parallax, or $r = \frac{1}{\sin \pi}$ its linear distance, and κ its semi-diameter expressed in semi-diameters of the Earth's equator. Further, let a d be the right ascension and declination of the Moon's centre for a spot on its surface in selenographical longitude λ and latitude β, N the node and J the inclination of the Moon's equator referred to the plane parallel to the Earth's equator, and α_0 the Moon's mean right ascension, analogous to l_0 its mean longitude ($\alpha_0 - \mathrm{N} = l_0 - \Omega + \Delta + 180$ in Encke's notation, adopted in the *Nautical Almanac*, N and J being the Ω' and i, v. *Monthly Notices*, vol. xli. p. 420 ff.) The formulæ for finding the coordinates of the spot λ β referred to the shadow axis are

$$\sec \beta \cos d \sin (a - \mathrm{N}) = \sin (\alpha_0 - \mathrm{N} + \lambda) \cos \mathrm{J} + \tan \beta \sin \mathrm{J}$$
$$\sec \beta \cos d \cos (a - \mathrm{N}) = \cos (\alpha_0 - \mathrm{N} + \lambda)$$
$$\sec \beta \sin d = \sin (\alpha_0 - \mathrm{N} + \lambda) \sin \mathrm{J} - \tan \beta \cos \mathrm{J}$$

$$x = r \cos \delta \sin (\alpha - \mathrm{A}) \qquad \xi = \kappa \cos d \sin (a - \mathrm{A})$$
$$y = r \sin (\delta - \mathrm{D}) + x \tan \tfrac{1}{2} (\alpha - \mathrm{A}) \sin \mathrm{D} \qquad \eta = \kappa \sin (d - \mathrm{D}) + \xi \tan \tfrac{1}{2} (a - \mathrm{A}) \sin \mathrm{D}$$
$$z = r \cos (\delta - \mathrm{D}) - x \tan \tfrac{1}{2} (\alpha - \mathrm{A}) \cos \mathrm{D} \qquad \zeta = \kappa \cos (d - \mathrm{D}) - \xi \tan \tfrac{1}{2} (a - \mathrm{A}) \cos \mathrm{D}$$

Hence the position angle p of the spot, its angular distance σ from the shadow axis and its linear distance r' from the Earth's centre are

$$r' \sin \sigma \sin p = x - \xi$$
$$r' \sin \sigma \cos p = y - \eta$$
$$r' \cos \sigma = z - \zeta.$$

If $\cos \epsilon_0$ is the proportion of the Earth's polar axis to the diameter of the equator

$$\sin \epsilon = \sin \epsilon_0 \cos D$$
$$\tan \epsilon' = \tan \epsilon \cos p$$
$$h = \frac{K - k}{R},$$

the radius of the section of the geometrical umbra is

$$r' \sin \sigma = \cos \epsilon' - h \,.\, r' \cos \sigma.$$

But the observed enlargement of the shadow proves that while a lunar spot near the border of the shadow receives still, or receives already, direct sunlight from a cusp outside the Earth's atmosphere, this illumination is so feeble that, as seen from the Earth, it merges into the dim illumination produced by the broken sunlight passing through the atmosphere and surrounding part of the Earth's disc. The uncertainty in fixing upon what ought to be considered the true border-line of the full shadow leaves it doubtful how much of the light near the margin of the Sun is to be treated as practically ineffective, or how much the value of K must be diminished in order to furnish the border-line. If I am not prevented from preparing ephemerides for physical observations of the Moon for 1898, I intend to give, for a number of lunar spots, predictions of the approximate times of their crossing the border-line of the shadow during the three lunar eclipses of that year, in the hope that many observers may join in making contributions for the closer examination of the question.

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The Orbit of δ Cygni. By S. W. Burnham.

The companion to δ *Cygni* was discovered by Sir William Herschel on 1783 September 20. The angle was measured, but all we know of the distance of the components at that time is found in his description of the appearance of the star in his reflector. Herschel says, "With 278 about $\frac{1}{2}$-diameter of L [large star]; with 460 $\frac{3}{4}$-diameter of L; with 932 full $1\frac{1}{2}$-diameter of L in hazy weather, which has taken off the rays of

L, and thereby increased the interval. Position, 18° 21′ *nf.*; perhaps a little inaccurate."

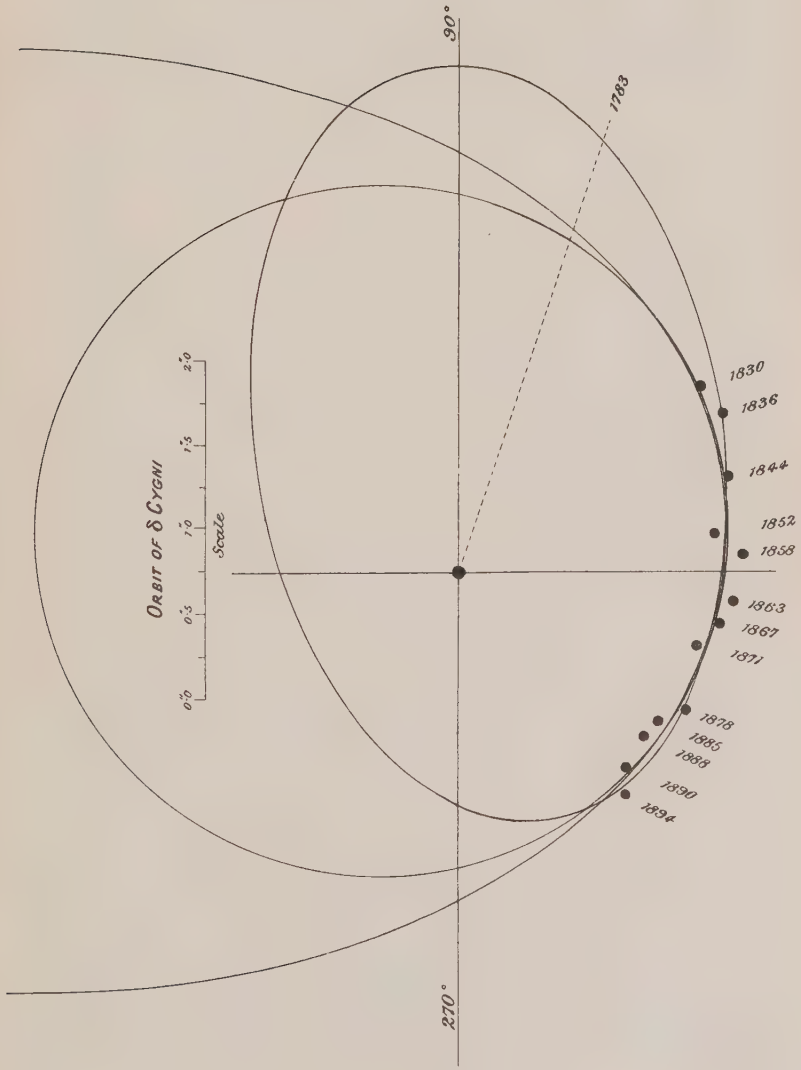
In the early part of the present century, Herschel examined this star several times, but could not see the companion. Referring to the matter at this time (*Phil. Trans.* 1804), he says :

"This double star, I believe, has furnished us with a second instance of a conjunction resembling that of ζ *Herculis*. The position, 1783 September 22, was 18° 21′ *nf.* 1802 January 3, 10 and 11, I could no longer perceive the small star, which must have been at least so near the large one as to be lost in its brightness. 1804 January 29, I examined this star with powers from 527 to 1500, and saw it as a lengthened star, but not with sufficient clearness to take a measure of its position. 1804 May 22, in a very clear evening, I tried 527 and 1500 with the 10-foot reflector, which acted remarkably well on the double stars, but I could not perceive the small star of δ *Cygni*. . . . A parallactic motion of δ will perfectly account for this occultation, for the situation of the two stars in 1783 was such that this motion must have carried the large star by this time nearly upon the small one."

It had been stated that Herschel was not able to see the companion at any time after his first observation, although he looked for it several times in the early years of this century ; but Sadler has examined his MS. now in the Royal Astronomical Society (*Observatory*, September 1886), and given extracts from it relating to his observations in 1802–3–4, from which it appears that this star was seen double at least once on 1803 May 28, with the 7-foot reflector, and noted, "near ½-diameter of smaller. The diameters are full 2 to 1."

These and other failures to see the companion, taken in connection with the opinion of Herschel that the star had actually become single, led to many erroneous ideas concerning the supposed rapid motion. Struve thought that more than a whole revolution had been performed between 1783 and the commencement of his observations in 1826, and that the period was less than 40 years.

It soon became apparent that the motion was very slow and practically uniform, and that there had been but little, if any, change in the distance from the time of discovery. The failures to see the companion were then frequently charged to variability in its light, and this notion is still entertained by some observers. Judging from my own experience with this and similar unequal pairs in the last twenty years, and considering the value of negative evidence in matters of this kind, there is nothing whatever to warrant even a suspicion that there has been at any time any sensible variation in the magnitude of the companion. Dawes was sceptical as to the alleged variability, and said that "great allowance must be made for the effect on such an object of different conditions of the atmosphere" ; and that he had


90°
1783
1830
1836
1844
1852
1858
1863
1867
1871
1878
1885
1888
1890
1894
270°
Orbit of δ Cygni
Scale
2·0
1·5
1·0
0·5
0·0

"never viewed this star under a moderately favourable state of the air without seeing the small companion plainly enough with apertures from $6\frac{1}{3}$ to $8\frac{1}{4}$ inches." In a pair of this class, the widely differing estimates of the magnitude of the smaller star by different observers is no evidence of any change in its brightness. Such double stars are frequently not seen with small apertures which under proper conditions show the companion well.

Four orbits have been computed for this pair in the last fifty years. The dates of the last measures used are given in the first column :—

1842	Hind	178·7 years	*A. N.* 565
1856	Behrmann	280·56 „	*A. N.* 1517
1865	Behrmann	415·11 „	*A. N.* 1561
1887	Gore	376·66 „	*Mon. Not.* L, 398

How few data were available in 1842, when the arc described from Struve down to that time was only about 15°, and there was nothing else but a rough single measure of the angle which did not purport to have any great value, is too obvious to require comment here. If Herschel's angle had been supplemented with the distance, and in both the highest accuracy had been reached which is attainable by a mean result from measures on a large number of nights by the best modern appliances, there still would have been nothing in the material to limit the arc representing these positions to a single ellipse, or to several wholly dissimilar ellipses. The material in 1856 for all practical purposes was no better, although the position-angle had changed a few degrees in the interval of fourteen years. Behrmann gave an ephemeris to 1880, and it is not surprising to find that it turned out, when the real position of the companion was determined, that the calculated place for the last-named date was 170° in error. Nine years later, with a few degrees of further change in the angle, he made a new investigation and obtained the second orbit given above, finding an entirely different ellipse, and increasing the period 135 years. This represents Herschel's angle of 1783 without error, and makes the distance at that time $2''{\cdot}56$. Gore's orbit also satisfies this angle within $0''{\cdot}7$, and makes the distance $2''{\cdot}72$.

On the accompanying diagram (Plate 10) I have given the angle of Herschel and thirteen repeatedly measured positions by leading observers at short intervals down to the present time. Of course many other measures might be included, but they would serve no purpose so far as the present question is concerned, and add nothing to the weight of the evidence bearing upon the orbit of this system. In fact, putting in all the measures, good, bad, and indifferent, would only exaggerate the uncertainty about this star, which is already sufficiently apparent from a use of the best material only.

I have shown on the diagram three possible orbits of this system, not with the idea or purpose of adding anything to our knowledge of the motion, but rather to emphasise the wholly indeterminate character of the problem presented. The arcs of the large ellipse and the circle are identical so far as the complete measures are concerned, and represent these positions as perfectly as they are ever likely to be represented, even when a complete revolution has been performed. The accuracy with which the measures are represented by these two entirely different orbits will appear from the following corrections to the observed positions.

	°	″		
1830·21	+0·1	−0·02	6*n*	Σ.
1836·52	+0·1	−0·09	4*n*	Σ.
1844·78	+0·8	−0·03	3*n*	OΣ.
1852·70	+1·3	+0·08	3*n*	OΣ.
1858·71	−1·3	−0·09	3*n*	OΣ.
1863·74	+0·7	−0·07	3*n*	OΣ.
1867·60	+0·2	−0·02	7*n*	Δ.
1871·69	−0·3	+0·05	9*n*	Δ.
1878·59	+2·5	−0·05	3*n*	Δ.
1885·73	−2·8	+0·06	4*n*	Hl.
1888·93	−3·4	+0·08	4*n*	Hl.
1890·57	+1·5	+0·03	4*n*	Hl.
1894·84	−0·8	−0·07	11*n*	Com.

These observations can also be represented precisely as well by an ellipse with the principal star outside altogether.

The smaller ellipse differs but little from the other figures, and for all practical purposes represents the measures equally well, since the largest distance variation does not exceed 0″·08. In this ellipse the distance in 1783 would be 2″·68, and the correction to the angle of Herschel is only 0°·5. And yet, aside from the short arc represented by the measures from 1830 to 1894, which is practically identical in all, these three orbits have nothing in common either as to the period or any other element.

The large ellipse gives a period of 1040 years, the circle 420 years, and the small ellipse 300 years. The true period may be anywhere between these limits, or perhaps outside of both. It must be remembered that in dealing with a short arc the agreement of observed and computed positions, however close and satisfactory it may be, does not even tend to prove that the orbit so found is correct, or anything like the real orbit. It is doubtful if the measures of another hundred years will be sufficient to determine the period of this pair to a reasonable approximation.

The Orbit of Sirius. By S. W. Burnham.

With the addition of the recent measures of the companion to *Sirius*, we have data for defining the orbit within narrow limits of error. Previous to this time all the orbits of this system have been based upon a short arc described by the companion from 37° to 85°, and this, with the errors of observation which are necessarily found in all micrometrical work, is of itself insufficient for even an approximate result, since the observed positions of so short an arc can be equally well represented by many ellipses having little in common. Most of the computers doubtless have been more or less influenced in the selection of the ellipse to represent the apparent orbit by the period of the theoretical disturbing body as deduced from the variable proper motion shown in the meridian observations. In this instance that would result in the adoption for the apparent orbit of nearly the smallest ellipse which would represent the short arc of the observed positions. Consequently, of the twelve orbits which have been computed since 1880, using the measures up to the respective dates, with periods from 49·5 to 58·5 years, only three of them exceed 52·2 years, and most of them are not more than a year or two from the theoretical time.

The companion was last measured previous to 1896 by the writer with the 36-inch refractor at the Lick Observatory in the early part of 1890. Satisfactory measures were secured on several nights although the companion was seen with considerable difficulty. In the latter part of the same year, and again in the following year, it was invisible under the best conditions ; and Barnard in the two or three succeeding years was unable to see the small star with the same instrument.

All subsequent attempts to see the companion at Mount Hamilton and elsewhere with the largest telescopes down to 1896 October, resulted in failure. Two observers at the Lick Observatory, Hussey and Campbell, have placed on record the invisibility of the companion with the 36-inch on six nights in 1896 February and March. As it is stated that the atmospheric conditions on each night were good, and *Sirius* near the meridian in all the observations, it is probable that this failure to see the small star was due to the inexperience of the observers in double-star work, since it was easily found by Aitken with the same instrument less than eight months later, when the distance could not have increased more than 0″·2. Any form of orbit which would permit of a rapid increase in the distance at this point would be inconsistent with the prior measures.

The arc passed over by the companion from 1890 to 1896 is 170°, so that altogether we have a path of 255° for the motion of the companion since its discovery in 1862 by Alvan G. Clark ;

and this should be sufficient for the determination of the elements of the orbit with only a small percentage of error.

In my papers on *Sirius* in *Monthly Notices* for 1891 April and November, and 1893 June, I have collected and arranged in chronological order all the measures of the companion from its discovery down to my last measures in 1890; and from these observations have deduced twenty-nine yearly means for the position of the companion for each year from 1862 to 1890. These means were made with all possible care, and with due reference to the probable relative value of the individual sets of measures, based upon the number of nights, the experience of the observer in micrometrical and double-star work, the instrument used, and the apparent general value of the several series of measures when considered as a whole. This was all done in advance of any use of the observations for obtaining the orbit, and therefore wholly independent of any prejudice or theory concerning the period or anything else. From these yearly positions I found the apparent orbit given in the first paper referred to. The complete elements (*Mon. Not.* liii. 482; *Pub. Lick Observatory*, ii. 54, 239) are as follows:—

$P = 51{\cdot}97$ years.	$i = 50°{\cdot}8$
$T = 1893{\cdot}5$	$\Omega = 40°{\cdot}3$
$e = 0{\cdot}568$	$\lambda = 135°{\cdot}4$
$a = 8''{\cdot}31$	

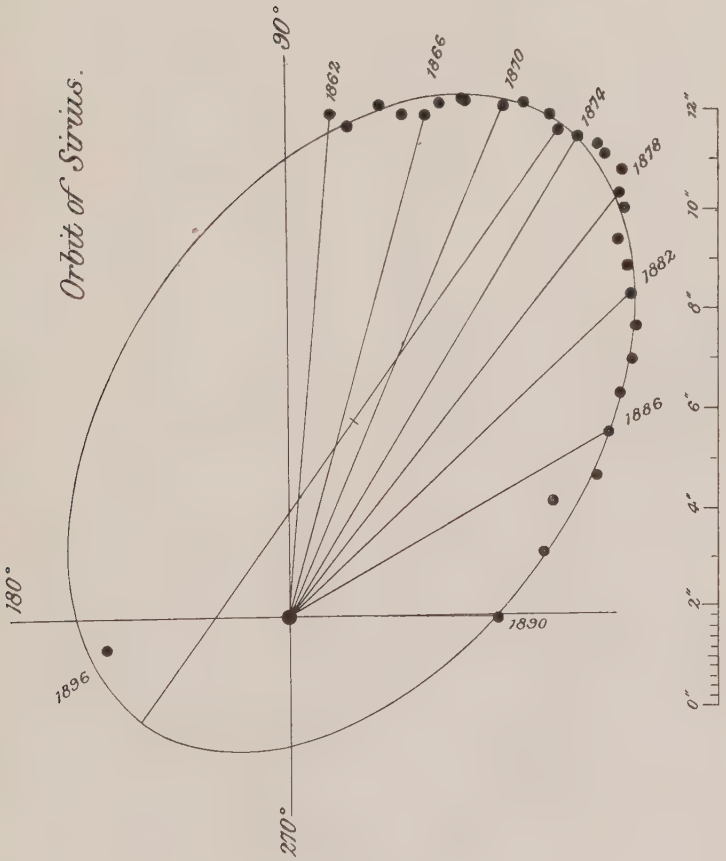
The fact that the companion could not be seen with the 36-inch in 1891, only six months after it had been measured in the early part of that year, was a certain indication that the form of the orbit in the fourth quadrant must be such as to require a rapid diminution in the distance of the companion at that point; and therefore, as stated in my paper at the time, nearly the smallest ellipse that would properly represent the measures was adopted as the most probable orbit. But for this consideration there was nothing to justify the investigation at all from such data, since an orbit with a period of several years more would have been equally probable, and would have represented the measures down to that time equally well.

The accompanying diagram (Plate 11) shows the yearly positions referred to, and the apparent orbit now determined from a careful consideration of all the available data. The only addition to the measures used in the former orbit is the following position derived from the measures on six nights by Aitken and Schaeberle, at Mount Hamilton:—

1896·85	189°·3	3″·71

From this ellipse I find the following elements:—

$P = 51{\cdot}80$ years.	$i = 44°{\cdot}7$
$T = 1893{\cdot}77$	$\Omega = 37°{\cdot}2$
$e = 0{\cdot}600$	$\lambda = 135°{\cdot}7$
$a = 7''{\cdot}62$	


Orbit of Sirius.
90°
180°
270°
1862
1866
1870
1874
1878
1882
1886
1890
1896
12″
10″
8″
6″
4″
2″
0″

Apparent orbit :—

Length of major axis = 14°·48
Length of minor axis = 9″·32
Angle of major axis = 54°·4
Angle of periastron = 251°·9
Star from centre = 4″·09

In the following table will be found the corrections to the observed positions as required by the final orbit :—

	°	″		°	″
1862·20	−0·5	−0·20	1877·18	+0·4	−0·30
1863·22	−1·4	+0·14	1878·12	0·0	−0·07
1864·20	−0·2	−0·10	1879·02	−0·2	−0·09
1865·22	0·0	+0·20	1880·18	−1·0	+0·08
1866·21	+0·2	+0·30	1881·19	−0·5	+0·01
1867·17	−0·4	+0·18	1882·12	0·0	+0·01
1868·16	0·0	−0·01	1883·14	+0·6	−0·01
1869·17	−2·0	+0·17	1884·22	−0·1	0·00
1870·15	−0·2	+0·04	1885·19	−0·8	−0·08
1871·21	0·0	−0·01	1886·14	+0·6	+0·04
1872·22	+0·8	−0·10	1887·19	+0·5	−0·09
1873·21	+0·3	+0·12	1888·24	−4·1	+0·18
1874·19	+0·08	+0·02	1888·97	−0·1	+0·03
1875·22	+0·7	−0·18	1890·27	0·0	0·00
1876·22	+0·3	−0·19	1896·85	+1·3	+0·29

It will be seen that this differs but little from the orbit previously found. The change in the period is less than two-tenths of a year. Aside from the position of the companion in 1896 as required by this orbit, I have no doubt that the distance of that year was undermeasured, and that it will be found hereafter that the above tabular correction is not too large. Remembering the appearance and difficulty of the companion in 1890 with the same instrument, when the distance could not have been much less than 4″·2, and the fact that I failed to see the companion at all a few months later, it appears to be improbable that the distance in 1896 was less than 4″.

The following is a short ephemeris of the companion :—

	°	″		°	″
1898·20	172·5	4·4	1900·20	151·0	4·9
1899·20	161·0	4·6	1901·20	141·1	5·2

With the extended arc given by the measures of 1896, there is but little latitude in the selection of the apparent ellipse. Of course, everything depends upon the substantial accuracy of the

last position, and especially with reference to the angle, because all the preceding positions can be represented by other ellipses. If the correctness of this position is confirmed by future measures, as it doubtless will be, there can be little change in these elements.

A New Quadruple Stellar System. By R. T. A. Innes.

The star κ *Toucani* was discovered to be double by Sir John Herschel on the night of 1834 November 3 with his 18-inch reflecting telescope. He remarks, "Very beautiful, excellent and careful measures with power 320 and triangular aperture."

His reflector measures are—

1834·84	12°·0	2″	6½ and 9
1834·90	11·3	2½	6 „ 9

He measured it again in 1836 with his 5-inch refractor, getting

1836·64	17°·8	5″·78	5½ and 11
1836·91	14·2	4·24	6½ „ .10

Amongst later measures may be cited

1871·9	0°·0	5″·19	H. C. Russell, Sydney.
1894·9	354·8	5·52	John Tebbutt, Windsor, N.S.W.

The comparison is also to be found in the Cordoba General Catalogue, being called 7th magnitude there.

The magnitudes may be taken as 5·6 and 7·8, the companion being of a purple colour.

The above measures alone would not prove physical connection, but the large proper motion of κ *Toucani* 0″·407 towards 86°·2 puts it beyond doubt.

The star Lac. 353, mag. 7·3, is 5′ 20″·0 north preceding κ *Toucani*. Whilst observing it in 1895 with a reflecting telescope made by Mr. F. Dixon Edmonds, an amateur, residing in Sydney, I noticed that it was also a double star (No. 27 of the list given on p. 262 of vol. vi. of the *Journal of the British Astronomical Association*).

An estimate made at that time gave

1895·8 180° 1″ ±

and one made here with the 7-inch refractor is

1896·8 185° 1″·4 ±

magnitudes 7·7 and 8·7.

I have since found that this pair shares in the proper motion of κ *Toucani* and its companion.

As the older catalogues are not very accurate I have preferred to exhibit the results given in this way. From the recorded positions of Lac. 353 I have subtracted the positions given in the same catalogues of κ *Toucani*, getting for the equinox of 1900.

		Δ α	Δ δ	Proper Motion of κ *Toucani* since 1752. α	δ
		s	′ ″	s	″
Lacaille	1752	−46·57	+3 46·1	+ 0·00	+ 0·0
Brisbane	1825	−47·37	3 15·9	5·53	6·1
Cape	1835·8	...	3 24·1	6·34	7·0
Gilliss	1851·9	−46·24	3 21·9	7·57	8·4
Cape	1874·8	−46·59	3 23·9	9·30	10·2
Cordoba	1875·0	−46·32	3 23·4	9·31	10·3
	1896·9	−46·71	3 23·1	10·97	12·1

The differences for 1896·9 result from a comparison with κ *Toucani* with the 7-inch equatorial, Lac. 353 being found in no later catalogue than the C. G. Arg.

The proper motion of κ *Toucani* used is +0ˢ·0757 in R.A. and +0″·0835 in Dec.; in this determination, which is a preliminary one, recent Cape meridian observations have been included.

If Lac. 353 had no proper motion its distances from κ *Toucani* should have been as follows—starting from 1752 :—

	Δ α	Δ δ		Δ α	Δ δ
	s	′ ″		s	′ ″
Lacaille	−46·57	+3 [46·1]	Cape	−55·89	+3 13·7
Brisbane	52·90	3 [9·8]	Cordoba	55·63	3 13·1
Cape	...	3 17·2	1896·9	57·68	3 11·0
Gilliss	53·81	3 13·6			

Hence we may reasonably infer that the pairs κ *Toucani* and Lac. 353 are physically connected. I know of no similar system.

The circumstances of these stars give some hopes of a measurable parallax.

Finally, it will be obvious that when the periods of both pairs are known, the common period of the system will be found as soon as the direction of motion of one pair around the other is known.

Royal Observatory, Cape of Good Hope:
1897 *February* 24.

LEEDS PHILOSOPH
AND LITERARY

On the determination of the Epoch-Correction of an adopted system of Right Ascensions of Clock Stars from observations of the Sun, including the effects of personalities; and applications of the results to the determination of the errors of the Tables of the Sun and Moon. By E. J. Stone, M.A., F.R.S., Radcliffe Observer.

There are few points in practical astronomical work which involve more delicate considerations than those connected with the determination of the absolute corrections required by the right ascensions of our Clock Stars, and the determination of the errors of the Tables of the Sun in longitude. I propose to give in the present paper some results on this subject to which I have been led, and some practical applications.

In an investigation of this kind it is necessary to accurately define the physical meanings of the symbols employed; and the notation which I have adopted is as follows:—

θ, λ, ω denote the longitude and latitude of the Sun's centre, and the "Obliquity of the Ecliptic," computed from a set of Solar Tables with an assigned value of the variable, t.

$\theta+\delta\theta$, $\lambda+\delta\lambda$, $\omega+\delta\omega$ denote the corresponding exact values.

α, Δ denote the corresponding tabular R.A. and N.P.D. computed in the usual way from the values of θ, λ, ω.

α', Δ' are the R.A. and N.P.D. of the Sun's centre as found directly from observation; and to which the variable, t, with which the tabular places are computed, is assumed to correspond.

$\alpha'+x$, $\Delta'+y$ are the exact values of the R.A. and N.P.D.

θ', λ' are the longitude and latitude of the Sun's centre from observation as found from α', Δ', and ω.

$\theta'+\delta\theta'$, $\lambda'+\delta\lambda'$ are the exact values of the longitude and latitude of the Sun's centre.

We have, therefore, $\theta+\delta\theta=\theta'+\delta\theta'$; $\lambda+\delta\lambda=\lambda'+\delta\lambda'$ identically.

The systematic errors in R.A., x, with which we shall be principally concerned arise from

(1) Errors in the adopted R.A. of the clock stars.

(2) Differences in personalities in the observations of transits of stars and limbs of the Sun.

The systematic errors in N.P.D., y, will be chiefly due to

(3) Errors in the adopted latitudes of the observing stations.

(4) Uncorrected division-errors of the circles.

(5) Personalities on the part of the observers.

(6) Errors of the adopted refraction tables.

We shall neglect the squares and products of the small angles $\delta\theta$, δ, λ, $\delta\lambda$, $\delta\omega$, x, and y ; and we then have

$$(7)\quad \alpha' + x = \alpha + \frac{\cos\omega}{\sin^2\Delta}\,\delta\theta - \frac{\sin\omega}{\sin^2\Delta}\sin\theta\,.\cos\theta\,.\,\delta\omega - \frac{\sin\omega}{\sin^2\Delta}\,.\cos\theta\,.\,\delta\lambda,$$

$$(8)\quad \Delta' + y = \Delta - \frac{\sin\omega}{\sin\Delta}\,.\cos\theta\,.\,\delta\theta - \frac{\cos\omega}{\sin\Delta}\,.\sin\theta\,.\,\delta\omega - \frac{\cos\omega}{\sin\Delta}\,.\,\delta\lambda,$$

or, putting $\alpha - \alpha' = f$, $\Delta - \Delta' = e$, we have

$$(9)\quad x = f + \frac{\cos\omega}{\sin^2\Delta}\,.\,\delta\theta - \frac{\sin\omega}{\sin^2\Delta}\,.\sin\theta\,.\cos\theta\,.\,\delta\omega - \frac{\sin\omega}{\sin^2\Delta}\,.\cos\theta\,.\,\delta\lambda,$$

$$(10)\quad y = e - \frac{\sin\omega}{\sin\Delta}\cos\theta\,.\,\delta\theta - \frac{\cos\omega}{\sin\Delta}\,.\sin\theta\,.\,\delta\omega - \frac{\cos\omega}{\sin\Delta}\,.\,\delta\lambda,$$

where $\frac{f''}{15}$ and e'' will be the errors of the Solar Tables in R.A. and N.P.D., as found directly from observation.

It is these tabular errors in R.A. and N.P.D., and the errors in longitude and latitude deduced from them, which are given in the published results of the Observatories at which solar observations are made.

The values of x, y, $\delta\omega$, $\delta\lambda$, and $\delta\theta$ have to be found from a discussion of equations of the form (9) and (10), or from others which are deducible from them ; and one of the chief practical difficulties of the problem of finding these and the other constants involved in the expressions of the solar coordinates is connected with the necessity for the separation of any constant correction, x, required by the observed R.A. from any existing constant error, or slowly changing errors, in the Solar Tables in longitude. The equations

$$(11)\quad \delta\lambda + x\sin\omega\,.\cos\theta + \delta\omega\sin\theta + y\frac{\cos\omega}{\sin\Delta} = f\,.\sin\omega\,.\cos\theta + e\frac{\cos\omega}{\sin\Delta},$$

$$(12)\quad \delta\theta = -f\,.\cos\omega + e\frac{\sin\omega}{\sin\Delta}\,.\cos\theta + x\cos\omega - y\frac{\sin\omega}{\sin\Delta}\,.\cos\theta,$$

which are directly deduced from (9) and (10), will be found more convenient for our purpose than those equations.

If the Errors of the Tables in Ecliptic Polar Distance and Longitude, as deduced from the observed R.A. and N.P.D., and the tabular obliquity are denoted by E and L respectively, we have

$$\mathrm{E} = f\,.\sin\omega\,.\cos\theta + e\frac{\cos\omega}{\sin\Delta};\quad \mathrm{L} = f\cos\omega - e\frac{\sin\omega}{\sin\Delta}\,.\sin\theta;$$

and, therefore,

$$(13)\qquad \delta\lambda + x\,.\sin\omega\,.\cos\theta + \delta\omega\,.\sin\theta + y\frac{\cos\omega}{\sin\Delta} = \mathrm{E},$$

$$(14)\qquad \delta\theta = -\mathrm{L} + x\,.\cos\omega - y\frac{\sin\omega}{\sin\Delta}\,.\cos\theta.$$

The equations of the form (13) are independent of $\delta\theta$; and because such is the case we are able to separate the constant, x, from any existing constant error of the Solar Tables in longitude by finding x from (13). We need not discuss here the cause or causes which may give rise to such errors.

The necessity of treating the equations (9) and (10) simultaneously in order to determine x and $\delta\vartheta$ has been frequently overlooked; and erroneous results have sometimes been obtained by the violation of this condition. If we denote the mean error in longitude of the Solar Tables in use for the observations of the year by $\delta\theta_0$; the use of the equation (9) independently of (10) would give sensibly the value $x' = x - \frac{\delta\theta_0}{\cos\omega}$ instead of x; and the use of x' instead of x in the equation (14) would give the errors of the Tables in longitude sensibly cleared from any existing mean error, $\delta\theta_0$, whatever Solar Tables were in use: for

$$\delta\theta = -\mathrm{L} + x\cos\omega - y\frac{\sin\omega}{\sin\Delta}\cdot\cos\theta = -\mathrm{L} + x'\cdot\cos\omega + \delta\theta_0 - y\frac{\sin\omega}{\sin\Delta}\cdot\cos\theta$$

$$\therefore\ \delta\theta - \delta\theta_0 = -\mathrm{L} + x'\ \cos\omega - y\frac{\sin\omega}{\sin\Delta}\cdot\cos\theta.$$

Before, however, we can separate the angles $\lambda+\delta\lambda$ and $\omega+\delta\omega$, we have to define what is meant by the plane of the Ecliptic.

The Ecliptic in practical astronomical work is selected so that the Sun's centre merely oscillates above and below that plane; and the expression for the Sun's latitude $\lambda+\delta\lambda$, when measured from the "Ecliptic," is an exceedingly small angle which contains no terms of the form

$$\mathrm{C} + \mathrm{A}\sin\theta + \mathrm{B}\cdot\cos\theta \qquad (15)$$

where C, A, and B are *constants*.

The small changes of latitude expressed by terms of this form, and the small secular changes which exist when the plane of reference is a fixed plane, are in practice thus thrown upon the longitude of the node and inclination of the moving plane of reference, the "Ecliptic."

On account of the importance of the equations (13) and (14), it may perhaps be worth while to obtain them directly without the introduction of the errors of the Tables in R.A. and N.P.D. We have with the adopted notation

$$\sin(\lambda'+\delta\lambda') = \cos(\omega+\delta\omega)\cos(\Delta'+y) - \sin(\omega+\delta\omega)\sin(\Delta'+y)\cdot\sin(\alpha'+x) \qquad (16)$$

$$\tan(\theta'+\delta\theta') = \sin(\omega+\delta\omega)\cot(\Delta'+y)\cdot\sec(\alpha'+x) + \cos(\omega+\delta\omega)\cdot\tan(\alpha'+x) \qquad (17)$$

$$\sin\lambda' = \cos\omega\cdot\cos\Delta' - \sin\omega\cdot\sin\Delta'\sin\alpha' \qquad (18)$$

$$\tan\theta' = \sin\omega\cdot\cot\Delta'\cdot\sec\alpha' + \cos\omega\cdot\tan\alpha'. \qquad (19)$$

In these equations the only tabular quantity is ω.
These equations lead at once to the following

$$(20) \qquad \delta\lambda' = -\sin\omega \,.\, \cos\theta' \,.\, x - \delta\omega \,.\, \sin\theta' - y\frac{\cos\omega}{\sin\Delta'}$$

$$(21) \qquad \delta\theta' = x\cos\omega - y\frac{\sin\omega}{\sin\Delta'} \,.\, \cos\theta'.$$

But we have $\lambda+\delta\lambda=\lambda'+\delta\lambda' : \theta+\delta\theta=\theta'+\delta\theta'$ identically.
We can find λ' and θ' from (18) and (19), and then by a direct comparison between them and the tabular quantities λ and θ, we have

$$\theta-\theta' = \mathrm{L}, \text{ and } \left(\frac{\pi}{2}-\lambda\right)-\left(\frac{\pi}{2}-\lambda'\right) = \lambda'-\lambda = \mathrm{E}$$

$$\therefore \delta\lambda' = \delta\lambda - \mathrm{E} \qquad\qquad \delta\theta' = \delta\theta + \mathrm{L};$$

and, as before,

$$(13) \qquad \delta\lambda + x \,.\, \sin\omega \,.\, \cos\theta' + \delta\omega \,.\, \sin\theta' + y\frac{\cos\omega}{\sin\Delta'} = \mathrm{E}$$

$$(14) \qquad \delta\theta = -\mathrm{L} + x\cos\omega - y\frac{\sin\omega}{\sin\Delta'} \,.\, \cos\theta'.$$

In the equation (14) the term $x\cos\omega$ introduces a constant error into our determinations of the corrections required by the Tables in longitude, and the term $-y\frac{\sin\omega}{\sin\Delta}\cos\theta'$ affects the determination of the longitude of the perigee. It would appear that these corrections, if determinable with any close approximation to their true values, should be applied before attempts are made to correct the errors of the Tables.

When the equation (13) is thus found, it perhaps brings more clearly before the mind that, since the expression for the latitude $\lambda+\delta\lambda$ contains no terms of the form (15), the errors in Ecliptic Polar Distance, E, can contain the effects of no terms of this form except those due to the errors in R.A., N.P.D. and obliquity, x, y and $\delta\omega$; and that any required *constant* corrections x, $\delta\omega$, and y can thus be found from the equation

$$(22) \qquad x\sin\omega\cos\theta + \delta\omega \,.\, \sin\theta + \frac{y\cos\omega}{\sin\Delta} = \mathrm{E}$$

by confining our attention to the effects of terms of the required form and suitably grouping the errors in Ecliptic Polar Distance; but the existence of any sensible errors, $\delta\lambda$, will to some extent increase the errors of the determinations of the constants x, $\delta\omega$, and y as thus found from observation.

If all the observations during a year were made by the same person, we might assume without serious error that x and y would be constants so far as the personalities of the observer were concerned: and, therefore, if we included the mean effects of the errors of the refraction tables and uncorrected division-errors with those of the personalities of the observer in N.P.D.,

we should have x and y constants for all the observations of a year.

A discussion of the equations (22) when the observations of the Sun were fairly distributed over the year would then lead to fairly approximate values of the constants x, $\delta\omega$, and y. But, on account of x being multiplied by a small factor 0·398 ; the existence of small errors, $\delta\lambda$, which have been neglected ; and the magnitude of the chance errors of observation in the case of the Sun ; the probable errors of the determinations of the important constant, x, will not, even in the case of a single observer, be inconsiderable.

But in practice it is difficult to secure the condition that all the observations of the year should be made by the same person, and if more than one person be engaged on the observations we have to meet the difficulty that the constants, x, are not the same for all observers.

The plan which has been followed at Greenwich for many years, and which I have adopted at Oxford, has been to ignore the differences between the constants x and y due to personal peculiarities ; and thus to obtain an approximation to the average values of the constants for the observers, and of the mean error of the Solar Tables in longitude. If the observations were thoroughly well distributed amongst the different observers over the whole year, and numerous enough to destroy the effects of chance errors of observation, the results thus found would be close approximations to the truth ; but in practice the required distribution is rarely secured, and the probable errors of the determination of the required constants for each year are thus greatly increased. The effects upon the errors of the tables in longitude of the discordances between the values of x thus found for different years without change of instrument, right ascensions of clock stars, or observers, are comparable in magnitude with the errors of the Solar Tables now in use ; and, because such is the case, it has not been usual to directly apply the corrections x, y, and $\delta\omega$, found for each year, to the errors in ecliptic polar distance and longitude deduced from the uncorrected observations in R.A. and N.P.D. But the means corrections for $\frac{x''}{15}$ found from the observations of several years are applied as a correction to the R.A. of the fundamental stars every few years when an independent standard catalogue is formed ; corrections for $\delta\omega$ are applied at intervals when the correction has become sufficiently large to render this desirable, and the effects of the constant y are usually neglected.

This method of dealing with the difficulty certainly prevents any serious accumulation of errors either in the right ascensions of the stars or in the errors of the solar tables ; but it can hardly be considered as perfectly satisfactory.

The personal differences in observing transits of stars and of the Sun between the three observers at Oxford happen to be

well marked. This affords facilities for testing the practicability of separating the personalities of the observers from the errors of the solar tables; and I have, therefore, separated the observations of the Sun made by Messrs. Wickham, Robinson, and Bellamy, 1884–1891. The values of the constants thus found for each observer from the observations of the different years must be expected to be rather discordant from the paucity of the observations and the unfavourable distribution of the observations in some of the years; but the mean results for several years should be fairly reliable. At all events, I have thought the point worth a trial, and the results will be found in Table (*a*).

Radcliffe Observations of the Sun, 1884–1891.

Table (*a*).

Year.	No. of Obs.	(x . sin ω).	$\delta\omega$.	y.	δa.	Error of Longitude (Tabular *minus* Observed),	Error of Longitude (Tab.−Obsd.) corrected for mean values of x for each observer.
				Observer W.			
		"	"	"	s	"	"
1884	23	+0·717	+0·785	−0·112	+0·120	+1·436	+0·422
1885	32	+0·349	+0·140	−0·226	+0·058	+1·641	+0·627
1886	24	+0·538	+0·342	−0·309	+0·090	+1·694	+0·680
1887	18	+0·552	+0·650	−0·507	+0·092	+2·064	+1·050
1888	13	−0·264	+0·718	−0·305	−0·044	−0·061	−1·075
1889	10	−0·176	+1·170	−0·061	−0·030	+1·475	+0·461
1890	20	+0·104	+0·693	−0·009	+0·017	+1·026	+0·012
1891	22	+1·094	+0·145	−0·383	+0·183	+2·150	+1·136
Weighted Means	162	+0·441	+0·497	−0·244	+0·074	+1·513	+0·499
				Observer R.			
1884	31	−0·032	−0·081	−0·296	−0·005	−0·033	+1·027
1885	27	−0·702	+0·532	−0·520	−0·117	−0·758	+0·302
1886	25	−0·282	+0·775	−0·135	−0·047	−0·702	+0·358
1887	21	−0·526	+0·198	−0·336	−0·088	−0·011	+1·049
1888	10	−0·466	−0·816	−0·182	−0·078	−0·048	+1·012
1889	26	−0·780	−0·155	−0·048	−0·131	−1·522	−0·462
1890	24	−0·814	+0·775	−0·325	−0·136	−1·403	−0·343
1891	30	−0·228	+0·158	−0·467	−0·038	−0·320	+0·740
Weighted Means	194	−0·461	+0·240	−0·302	−0·077	−0·632	+0·428

Year.	No. of Obs.	$(x \cdot \sin \omega)$.	$\delta\omega$.	y.	δa.	Error of Longitude (Tabular *minus* Observed).	Error of Longitude (Tab.—Obsd.) corrected for mean values of x for each observer.
				Observer F.B.			
		″	″	″	s	″	″
1884	33	+0·528	+0·146	−0·250	+0·088	+1·137	+0·270
1885	29	+0·815	+0·072	+0·100	+0·136	+1·801	+0·934
1886	20	+0·412	+0·004	−0·053	+0·069	+1·987	+1·120
1887	19	−0·407	−0·535	−0·267	−0·068	+1·627	+0·760
1888	12	−0·335	+0·203	−0·231	−0·056	+1·924	+1·057
1889	24	−0·312	+0·725	−0·222	−0·052	+1·384	+0·517
1890	11	+1·351	−0·157	+0·322	+0·226	+1·131	+0·264
1891	25	+0·813	+0·769	−0·560	+0·136	+2·127	+1·260
Weighted Means	173	+0·377	+0·197	−0·174	+0·063	+1·632	+0·765

The systematic character of the differences between the errors of the tables in longitude deduced directly from the observations of Messrs. Wickham, Robinson, and Bellamy is sufficiently pronounced to be recognised in the results of a single year. The mean errors of the tables in longitude for the years 1884–1891 are

(23) W. = +1″·513; R. = −0″·632; F.B. = +1″·632;

and the weighted mean is +0″·765.

The corresponding values of the constant, y, are

(24) W. = −0″·244; R. = −0″·302; F.B. = −0″·174.

These values of y agree fairly well with the weighted mean −0″·250, and they show that no very strongly marked personalities exist in the observations of the Sun in N.P.D.

The corresponding values of $\delta\omega$ are

(25) W. = +0″·497; R. = +0″·240; F.B. = +0″·197;

and these corrections to the tabular obliquity are also fairly accordant with the weighted mean +0″·280.

But the values of $(x \sin \omega)$ indicate the existence of well-marked differences in the habits of the observers in taking transits of stars and one, or both, of the limbs of the Sun.

The mean values of $(x \sin \omega)$ for the different observers are

(26) W. = +0″·441; R. = −0″·461; F.B. = +0″·377;

and for x

W. = +1″·108; R. = −1″·157; F.B. = +0″·946;

which show that the R.A. of the Sun as directly found by each observer requires to be corrected for

$$W. = +0^s{\cdot}074\,;\ R. = -0^s{\cdot}077\,;\ F.B. = +0^s\,063\,;$$

whilst the personal equations for transit observations of stars referred to the mean habit during the period under discussion are

$$W. = +0^s\,15\,;\ R. = -0^s{\cdot}37\,;\ F.B. = +0^s{\cdot}22.$$

It may be mentioned here that all the transit observations at the Radcliffe Observatory are made by the "eye-and-ear" method. The personal equations for stars are large, but the personal equation of Mr. McClellan, who succeeded Mr. Bellamy in 1892, falls about half-way between W. and R.

The differences in the errors of the solar tables in longitude for the different observers given in (23) are due to differences in personalities in observing transits of stars and of limbs of the Sun. When we apply the corrections, x, found for the three observers to their corresponding errors in longitude, see equation (14), we have for the mean errors of the tables in longitude, or $(-\delta\theta)$, at the epoch 1888·0,

$$(27) \qquad W. = +0''\,499\,;\ R. = +0''{\cdot}428\,;\ F.B. = +0''{\cdot}765$$

and the weighted mean is $+0''{\cdot}560$.

The extreme difference of the results (27) only amounts to $0''{\cdot}34$, whilst for the results (23) the extreme difference is $2''{\cdot}26$.

The results (27) are certainly within the chance errors of their determination, which include those of the constant $x \sin \omega$ multiplied by 2·3 ; whilst the results (23) are discordant beyond any possible chance errors of their determination or the effects of the errors of the tables in longitude. The personal corrections in time required by the right ascensions of the Sun for the different observers

$$W. = +0^s{\cdot}074\,;\ R. = -0^s{\cdot}077\,;\ F.B. = +0^s{\cdot}063$$

have been found quite independently of any existing errors of the tables in longitude ; and, although liable to sensible chance errors of determination, they must be regarded as approximate determinations of the absolute corrections required by the right ascensions of the Sun found by the three Oxford observers with the R.A. of clock stars in use.

If we apply these corrections to the observed R.A. of the different observers, the "errors of the tables," both in ecliptic polar distance and longitude, will be less dependent on the distribution of the observations during each year among the observers ; and we may be able to obtain more accurate values of the constants x', y, and $\delta\omega$ by a rediscussion of the errors of ecliptic polar distance. In this rediscussion the symbols y and $\delta\omega$ will denote the same physical quantities as before, but x' will not

denote the same quantity as x. The equations connecting the x and x' for the different observers are :
For

$$\text{W.}, x = +1''\cdot108 + x' ; \quad \text{R.} = -1''\cdot157 + x' ; \quad \text{F.B.} = +0''\cdot946 + x'.$$

In this rediscussion the constants x' will be assumed the same for each observer.

The results thus obtained for x', y, and $\delta\omega$ are given in the following table (β), and those found from the system hitherto adopted of neglecting the personalities are given in table (γ).

TABLE (β).

Year.	No. of Obs.	$(x'.\sin\omega.)$	$\delta\omega$.	y.	$\delta\alpha$.	Error of Long. (Tabular *minus* Observed).	Error of Long. (Tab.—Obsd.) corrected for mean values of x'.
		″	″	″	s	″	″
1884	87	+0·201	+0·204	−0·187	+0·034	+0·580	+0·594
1885	88	+0·092	+0·198	−0·186	+0·015	+0·632	+0·646
1886	69	+0·128	+0·452	−0·216	+0·022	+0·690	+0·704
1887	58	−0·174	+0·180	−0·440	−0·029	+0·953	+0·967
1888	35	−0·659	+0·122	−0·225	−0·110	+0·253	+0·267
1889	60	−0·551	+0·447	−0·166	−0·092	+0·085	+0·099
1890	55	−0·083	+0·560	−0·045	−0·014	−0·094	−0·080
1891	77	+0·432	+0·318	−0·451	+0·072	+1·021	+1·035
Weighted Means	529 = sum	−0·006	+0·308	−0·242	−0·001	+0·560	+0·574

TABLE (γ).

Year.	No. of Obs.	$(x.\sin\omega.)$	$\delta\omega$.	y.	$\delta\alpha$.	Error of Long. (Tabular *minus* Observed).	Error of Long. (Tab.—Obsd.) corrected for mean values of x.
		″	″	″	s	″	″
1884	87	+0·294	+0·260	−0·256	+0·049	+0·799	+0·624
1885	88	+0·233	+0·166	−0·157	+0·039	+0·961	+0·786
1886	69	+0·220	+0·450	−0·190	+0·037	+0·911	+0·736
1887	58	−0·100	+0·077	−0·422	−0·017	+1·169	+0·994
1888	35	−0·511	+0·034	−0·120	−0·086	+0·623	+0·448
1889	60	−0·557	+0·467	−0·224	−0·093	+0·140	−0·035
1890	55	−0·055	+0·463	−0·087	−0·009	−0·013	−0·188
1891	77	+0·509	+0·266	−0·468	+0·085	+1·180	+1·005
Weighted Means	529 = sum	+0·076	+0·280	−0·250	+0·013	+0·765	+0·591

The results of column 7 and not column 8 are usually given as the errors of the tables in longitude in the published results of the Observatories.

The results, Table (β), obtained by this rediscussion are in close agreement, as they should be, with those found by a separation of the observations of the Sun made by the different observers ; and they differ but little from those obtained by neglecting the differences between the values of x for the observers and trusting to the chance of a sufficiently uniform distribution of the observations amongst the observers over the year to destroy the effects of personality on the average values of the constants for all the observers.

The errors of Le Verrier's solar tables in longitude, or $-\delta\vartheta$ after the corrections for the mean values of x or x' have been applied (see equation 14), are as follows :—

Epoch 1888·0.

Table (α).—Weighted mean of separate individual results $= +0''\cdot560$.

Table (β).—Rediscussion with personal corrections $= +0''\cdot560 + 0''\cdot006 \times 2\cdot3 = +0''\cdot574$.

Table (γ).—Method hitherto adopted, but with correction for x applied $= +0''\cdot765 - 0''\cdot076 \times 2\cdot3 = +0''\cdot590$.

The close agreement of the results (γ), after correction for x, with (α) and (β) shows that for the years 1884–1891 the solar observations have been fairly distributed amongst the observers over the different parts of the orbit. If the constants x and y are neglected, the errors $+0''\cdot20 + 0''\cdot25 \cos\theta$, see equations 14, 24, and 27, are thrown upon the solar tables.

When the method of personal corrections is adopted, it will be necessary from time to time to examine these corrections by a rediscussion of the "errors of ecliptic polar distance" of the tables of the Sun deduced from the observations of the different observers.

This method of applying personal corrections, found independently of the errors $\delta\vartheta$, to the right ascensions of the Sun appears sound in theory, and advantageous in practice at those observatories where only two or three observers are engaged upon the work, and when the separation of x for each observer from the mean errors of the tables in longitude can, therefore, be practically effected.

There is one point connected with these differences of habit in observing the transits of stars and the limbs of the Sun which appears to deserve notice.

The corrections x which we have found refer strictly to the observations of the Sun in right ascension, and they will not be identical with those required by the adopted R.A. of the clock stars if the observer has different habits of observing the transits of the limbs of the Sun and stars. When, therefore, the epoch corrections are computed as usual from the equation

$$\delta\alpha^s = \frac{x''}{15^{s\cdot}}$$

such personalities of the observer, or observers, are necessarily thrown upon the fundamental right ascensions of the clock stars, and this is true whether all the observations of the Sun are made by one person or by several.

In the case of the three Oxford observers, the epoch-corrections thus computed for each observer are

$$W. = +0^s{\cdot}074\,;\ R. = -0^s{\cdot}077\,;\ F.B. = +0^s{\cdot}063.$$

The weighted mean is $+0^s{\cdot}013$, and this agrees very closely indeed with that which I used—viz. $+0^s{\cdot}014$—in the formation of the "Radcliffe Catalogue of 6,424 Stars for 1890." But this correction involves the effects of the mean personalities of the three observers as well as any constant error of the adopted right ascensions of the clock stars used for the determination of the errors of the clock.

We have no direct means, so far as I know, of deciding whether or no any particular observer has any personalities of the kind under consideration. We cannot, therefore, directly eliminate such effects of personality from our work. But we have a choice of methods. We can accept the mean habit of several observers as probably less affected by such personalities and more stable than the habit of any one observer, or we can select some standard observer and attempt to adopt his habits as free from error or as giving rise only to a constant error in our work. I am strongly of opinion that on fundamental points it is much safer to trust to the mean habit of several good observers than to that of one; in fact, the selected standard observer, practically, will change with age, and it will be exceedingly difficult to connect his past and present habits with those of other observers.

The solar observations discussed in the present paper afford, in my opinion, a rather striking example of the great reliability of means.

The observations at Oxford 1884–1891 were based on the Greenwich clock-star lists. The errors of the tables in longitude given by the separate observers, W., R., and F.B., uncorrected for personalities, differ systematically and very sensibly from each other; but if we take the mean error in longitude found directly from the observations of all the observers combined, both at Oxford and Greenwich, for the errors of the tables in mean longitude we have, 1884–1891:

Oxford	$+0''{\cdot}765$,
Greenwich	$+0''{\cdot}758$.

The two results are therefore practically identical. This is, of course, to some extent a matter of chance; but the errors of the tables given by the Oxford observations would have differed very sensibly from those given by the Greenwich observers had I rejected, underweighted, or corrected Mr. Robinson's observations

simply because his results differed from the observations of Messrs. Wickham and Bellamy. There is no sensible difference in the habits of the three observers W., R., and F.B. in taking transits of the first and second limbs of the Sun. To show that such is the case the corrections to the tabular semidiameter found from the observations of the different observers are given in Table (δ). The mean correction for the time of passage of the semidiameter is $+0^{s}\cdot063$, and this result does not differ by quite a hundredth of a second from the three separate results for the different observers. There are very slight indications of personal differences in the semidiameters in N.P.D.; but these differences are much too small and uncertain to render any attempts at correction necessary or desirable. But on the effects of personalities in the N.P.D. observations see the values of y given in Table (α).

Radcliffe Observations, 1884–1891.

Errors of Tabular Semidiameter of the Sun for each Observer.

Table (δ).

	R.A.						N.P.D.					
	W.		R.		F.B.		W.		R.		F.B.	
Year.		No. of Obs.		No. of Obs.		No. of Obs.		No. of Obs.		No. of Obs.		No. of Obs.
	s		s		s		″		″		″	
1884	+0·064	25	+0·069	34	+0·063	35	−0·36	26	+0·29	33	+0·49	39
1885	+0·074	30	+0·049	24	+0·067	25	+0·20	31	+0·28	26	+0·23	30
1886	+0·074	22	+0·065	26	+0·072	21	+0·34	22	+0·13	20	+0·29	23
1887	+0·057	18	+0·023	20	+0·080	14	+0·24	17	+0·29	18	+0·15	17
1888	+0·059	10	+0·045	8	+0·065	11	+0·54	11	+0·05	9	+0·53	12
1889	+0·050	10	+0·037	25	+0·076	23	−0·10	10	+0·11	25	+0·19	20
1890	+0·035	19	+0·067	24	+0·041	11	−0·06	16	+0·04	22	+0·33	9
1891	+0·072	22	+0·070	28	+0·086	23	+0·34	22	+0·34	27	+0·73	24
Weighted Means	+0·063	156	+0·056	189	+0·070	163	+0·13	155	+0·21	180	+0·38	174

The existence of well-marked differences in the observations of the transits of the Sun has induced me to try whether the same difference of habit exists in the case of the Moon. On account of the comparatively much larger errors of the Lunar Tables, and the rapidity with which they change, we cannot expect that the application of the corrections

$$W. = +0^{s}\cdot074 \qquad R. = -0^{s}\cdot077 \qquad F.B. = +0^{s}\cdot063$$

to the observed right ascensions will give the same mean errors in longitude for the different observers unless a large number of observations of the Moon, extending over many years, are included in the discussion. But with a large number of observa-

tions the effects of the real errors of the tables should not seriously affect the differences of the errors of the tables thus found for each observer. This test has been applied to the lunar observations made in the years 1884–1891, and the results thus obtained are given in the following Table (ϵ).

Radcliffe Observations of the Moon, 1884–1891.

Mean Errors of the Tables in Longitude for each Observer.

(*Tabular* − *Observed.*)

Table (ϵ).

Year.	Observer.	Errors of the Tables in Longitude.	Errors of the Tables in Longitude Corrected for Personal Values of x.	No. of Observations.
		″	″	
1884	W.	−0·15	−1·19	12
	R.	−3·21	−2·13	27
	F. B.	−0·30	−1·18	22
1885	W.	−1·19	−2·23	13
	R.	−3·85	−2·76	22
	F. B.	+0·25	−0·63	22
1886	W.	−1·65	−2·69	17
	R.	−3·49	−2·41	21
	F. B.	−0·30	−1·18	11
1887	W.	−3·17	−4·21	9
	R.	−4·19	−3·11	27
	F. B.	−0·99	−1·87	18
1888	W.	−1·49	−2·53	11
	R.	−3·41	−2·33	19
	F. B.	−1·45	−2·33	14
1889	W.	−2·91	−3·95	18
	R.	−4·56	−3·48	27
	F. B.	−2·11	−3·00	17
1890	W.	−2·21	−3·25	12
	R.	−4·89	−3·81	22
	F. B.	−2·45	−3·33	12
1891	W.	−1·45	−2·49	15
	R.	−4·18	−3·10	32
	F. B.	−1·95	−2·83	18
Weighted Means 1884–1891	W.	−1·79	−2·83	107
	R.	−3·99	−2·91	197
	F. B.	−1·07	−1·95	134

These results are the errors of Hansen's Tables, corrected for δt, as given in the volumes of the *Radcliffe Observations*, and with tabular semidiameters corrected by $-0^{s}\cdot06$.

It will be seen that the application of the personal equations deduced from the observations of the Sun has greatly reduced the differences of the errors of the Lunar Tables deduced from the observations of the Moon by the different observers.

As a further check upon the sensible uniformity of the habits of the observers in taking transits of the limbs of the Sun and Moon I have had the errors of the tables of the Moon in R.A. for both limbs collected for the different observers. The results uncorrected for personalities are given in Table (ζ), and the results corrected for the personalities, found from the observations of the Sun, are given in Table (η).

Mean Errors of the Tables of the Moon in R.A. for each Observer (Tabular − Observed), uncorrected for x.

TABLE (ζ).

Year.	☽1L. W. s	No. of Obs.	☽1L. R. s	No. of Obs.	☽1L. F.B. s	No. of Obs.	☽2L. W. s	No. of Obs.	☽2L. R. s	No. of Obs.	☽2L. F.B. s	No. of Obs.
1884	−0·068	9	−0·250	18	−0·066	14	+0·203	3	−0·142	9	+0·132	6
1885	−0·103	7	−0·293	15	−0·063	12	−0·070	5	−0·181	7	+0·126	10
1886	−0·142	13	−0·248	17	−0·070	5	+0·078	4	−0·040	3	+0·050	5
1887	−0·257	7	−0·326	18	−0·196	11	+0·055	2	−0·189	9	+0·180	6
1888	−0·175	6	−0·263	11	−0·066	9	+0·028	5	−0·175	8	−0·058	5
1889	−0·212	9	−0·383	15	−0·117	10	−0·160	7	−0·205	11	−0·114	7
1890	−0·267	7	−0·346	11	−0·201	9	+0·103	3	−0·324	11	+0·027	3
1891	−0·217	7	−0·337	20	−0·178	9	0·000	8	−0·197	11	−0·046	8
Weighted Means	−0·174	65	−0·305	125	−0·118	79	0·000	37	−0·199	69	+0·040	50

Corrected for Personal Values of x.

TABLE (η):

Year.	☽1L. W. s	No. of Obs.	☽1L. R. s	No. of Obs.	☽1L. F.B. s	No. of Obs.	☽2L. W. s	No. of Obs.	☽2L. R. s	No. of Obs.	☽2L. F.B. s	No. of Obs.
1884	−0·142	9	−0·173	18	−0·129	14	+0·129	3	−0·065	9	+0·069	6
1885	−0·177	7	−0·216	15	−0·126	12	−0·144	5	−0·104	7	+0·063	10
1886	−0·216	13	−0·171	17	−0·133	5	+0·004	4	+0·037	3	−0·013	5
1887	−0·331	7	−0·249	18	−0·259	11	−0·019	2	−0·112	9	+0·117	6
1888	−0·249	6	−0·186	11	−0·129	9	−0·046	5	−0·098	8	−0·121	5
1889	−0·286	9	−0·306	15	−0·180	10	−0·234	7	−0·128	11	−0·177	7
1890	−0·341	7	−0·269	11	−0·264	9	+0·029	3	−0·247	11	−0·036	3
1891	−0·291	7	−0·260	20	−0·241	9	−0·074	8	−0·120	11	−0·109	8
Weighted Means	−0·248	65	−0·228	125	−0·181	79	−0·074	37	−0·122	69	−0·023	50

The errors of the Lunar Tables are so large, and change so rapidly, that a complete elimination of their effects from these results can hardly be expected. But the much closer agreement of the corrected results for the different observers is quite evident.

If we take the means of the results for the two limbs for each observer to eliminate the effects of the tabular semi-diameter, the correction to which is not very accurately determined, we have

$$W. = -0^{s}\cdot161\,;\quad R. = -0^{s}\cdot175\,;\quad F.B. = -0^{s}\cdot102.$$

These results agree very closely ; but they appear to show, like the solar observations—see (27)—that the value of x found from Mr. Bellamy's observations in Ecliptic Polar Distance is rather smaller than the true value.

In the present paper attention has been directed to the existence of very sensible systematic differences in the habits of three skilled observers in taking transits of stars and of limbs of the Sun and Moon. The theory of the effects of such personalities has been discussed, and very approximate determinations of the values of these personalities have been found from a discussion of the solar observations, independently of any errors of the Solar Tables in longitude ; and it has been shown that no sensible differences in the habits of the observers exist in taking transits of the two limbs of the Sun. These personal corrections have been shown to be applicable to the case of the Moon as well as of the Sun. The errors of the Solar and Lunar Tables are, by the adoption of these corrections, sensibly freed from the effects of the personalities in the observers ; and they differ sensibly from those which are usually accepted as the errors of the tables when the corrections, x, are neglected.

It would appear that a very sensible increase in the accuracy of our determinations of the errors of Solar and Lunar Tables should result from this method of finding the personal corrections, x, and their use as corrections to the observed R.A.

Preliminary Note on a Personal Equation depending on Magnitude affecting the Right Ascensions of the Stars in the Cambridge Zone Catalogue of the Astronomische Gesellschaft, and its determination from Astrographic Catalogue Plates. By Arthur R. Hinks, B.A.

(*Communicated by Sir R. S. Ball.*)

The Cambridge Zone Catalogue was completed by Mr. Graham at the end of 1896. In accordance with the programme of the Gesellschaft the Cambridge zone overlaps the northern Berlin zone (Dr. Becker's) by 20′. There are thus a number of stars common to the two catalogues, and a comparison between their places in the two affords a means of determining any systematic error which is not common to both.

With the assent of the director I undertook to make the comparison as part of my regular work at the Cambridge Observatory, and the results of this and the subsequent work suggested by it form the substance of this note.

The zone $+24^\circ\ 50'$ to $+25^\circ\ 10'$ is common to the Cambridge and Berlin zones, and 671 stars, mainly within these limits, are found in both catalogues. A table was formed giving the difference (Cambridge − Berlin) of the catalogue right ascensions and declinations of each star. It was immediately evident that the differences in R.A. were small, about $0^s{\cdot}07$ on the average, but were in general positive, and that there was thus a systematic difference between the catalogues. The differences in declination were small, about $0''{\cdot}5$ on an average, and with them we are not concerned in this note. They appeared to be mainly accidental.

The most probable cause of such a discrepancy appeared to be the existence of a personal equation depending on magnitude affecting one or more probably both series of observations. Several of the observatories co-operating in the scheme of the Astronomische Gesellschaft had already investigated this question in reference to their own observations, and determined a magnitude equation of sensible amount. The Cambridge Catalogue is particularly well adapted to furnish evidence of value on this point, since by far the greater part of the R.A. observations were made by one and the same observer, Mr. Graham. The results should, therefore, be homogeneous so far as personal equation is concerned.

Before, however, discussing the differences with reference to personal equation, it is necessary to satisfy ourselves as far as possible that no other causes may have contributed to the observed discrepancy. To do this we must glance at the circumstances under which the observations were made.

The work on the zone catalogue was begun at Cambridge in the summer of 1872. Until the autumn of 1883 the stars were

observed in zones, mostly over three wires, and they were reduced by means of tables prepared for each zone. The stars which then remained unobserved were not sufficiently well distributed to make zone observations desirable. The zone method was therefore dropped, and thenceforward the catalogue stars were observed and reduced in the same manner as the clock stars, except that they were usually taken over five wires instead of seven.

The Berlin observations were all made in zones, and were completed in the four years 1880–1883.

From 1883 January 1 a small and probably quite unimportant change was made in the adopted clock-star list. Up to this date the Astronomische Gesellschaft had published yearly star ephemerides based upon Dr. Auwers' *Fundamental-Catalog*, and the Cambridge R.A.'s depended upon these. After 1882 these ephemerides were discontinued, and at the same time the clock-star list of the *Berliner Jahrbuch* was much enlarged and the star places made to depend upon the *Fundamental-Catalog*. A number of the stars in the latter were, however, omitted from the ephemeris as superfluous or otherwise undesirable as clock stars, and some corrections to the star places which had been published in the *Vierteljahrsschrift der Astronomischen Gesellschaft* were incorporated. From 1883 January 1 the right ascensions of the Cambridge stars are based upon this list.

In order to determine whether the trifling dissimilarity in the circumstances of the Berlin and Cambridge work, and the slight want of homogeneity in the latter, had had any appreciable effect on the results, the observed differences of R.A. (Cambridge — Berlin) were weighted and divided into three groups according to the dates of the individual observations at Cambridge.

Group I. included all stars of which the individual observations were all taken before 1884 January 1, and whose catalogue R.A.'s depend, therefore, almost entirely upon zone observations and reductions and upon the Gesellschaft ephemerides.

Group II. included all stars of which the individual observations were made some before and some after 1884 January 1.

Group III. included all stars of which the individual observations were all made after 1884 January 1. They were neither observed nor reduced by zones, and their R.A.'s depend upon the *Jahrbuch*.

The formula used for weighting the differences was

$$\frac{4mn}{m+n+\frac{1}{5}mn},$$

where m and n are the numbers of observations in the two catalogues. (See the papers by Messrs. Turner & Hollis, *Mem. R.A.S.*, vol. li. p. 53, and by Mr. Stone, *M.N.* 1869, p. 324.) It is to be remarked that this formula is properly applicable only when the probable errors of one observation in the two catalogues

are equal, which is not quite true in the present case. The error thus introduced will be, however, quite inappreciable.

The means for each group were taken for each three hours of R.A., and the results are given in the following table :—

TABLE I.

R.A. h h	I. Mean. s	Weight.	II. Mean. s	Weight.	III. Mean. s	Weight.	Totals. Mean. s	Weight.
0– 2	+·056	142	+·049	85	+·067	112	+·058	339
3– 5	+·063	148	+·082	77	+·100	229	+·089	434
6– 8	+·033	216	+·045	112	+·082	154	+·051	482
9–11	+·058	161	+·099	48	+·060	49	+·066	258
12–14	+·067	104	+·055	120	+·112	37	+·068	261
15–17	+·053	78	+·048	114	+·040	100	+·047	292
18–20	+·037	288	+·040	194	+·049	39	+·039	521
21–23	+·044	240	+·073	95	+·077	111	+·058	446
Totals ...	+·048	1377	+·056	845	+·078	831		

It is immediately evident from this table that there is a great part of the discrepancy between the two catalogues which cannot be attributed to the above-mentioned slight want of homogeneity in the Cambridge methods. The stars in Group I. were observed on the average four or five years earlier at Cambridge than at Berlin, but at both observatories the method of observation and reduction was the same. The resulting R.A.'s should therefore be strictly comparable. We find, on the contrary, that there is a well-marked discrepancy of about $^{s}\cdot05$ even in this first group. It does not seem probable, therefore, that the somewhat larger discrepancy shown by the third group can be due in any great degree to the small change in the method of observation and reduction which, as is mentioned above, took place in the later years of the work at Cambridge. If this change be supposed to have any influence at all, it can be measured only by the difference between the mean residuals in Groups I. and III.—that is, by about $^{s}\cdot03$. We have to account in some other way for the balance of the residual. But until we discover the cause of this residual as a whole we cannot be sure that the effect of such cause is the same in all three groups.

It is equally clear, from an inspection of the mean residuals for every three hours of R.A., that no large part of the discrepancy can be attributed to errors in the assumed positions of individual clock stars, such as might arise from erroneous determinations of their proper motion. It is true that there is a fairly well-marked minimum in hours 15–20, and perhaps also in hours 6–8 ; it is, however, as well marked in the early as in the late years, and can therefore scarcely be accounted for by a progressive error in the places of the clock stars. And the argument of the last paragraph applies here with equal force,

that, until we can account for the existence of the discrepancy as a whole, it is unsafe to assign any definite origin to variations in its amount.

It remains, then, to try whether the original hypothesis—of the existence of a personal equation depending on magnitude or illumination of the field—will account for the observed discrepancy. In the Cambridge meridian circle the arrangement designed to admit of the observation of faint stars over bright wires in a dark field was not successful. It was superseded by mounting a plate of red glass so that it could be lowered in front of the opening in the axis through which the light for the bright field illumination was admitted. The wires then appeared dark but faint in a dull red field. This glass was not used in the observation of clock stars, or in some cases of zone stars bright enough to be seen easily in the bright field. As a rule, stars below $7^{m}\cdot0$ were taken in the red field, and zone stars brighter than this sometimes in the red, sometimes in the bright field.

The magnitudes which have been adopted in this grouping are taken from the Berlin Catalogue. These are, in accordance with the programme of the Astronomische Gesellschaft, independent estimates on Argelander's scale. At Cambridge it was considered inadvisable to attempt more than a check on Argelander's Durchmusterung magnitudes. If a star differed notably from the magnitude there assigned to it the observer's estimate was noted. But in general the catalogue magnitudes are practically Argelander's. In view of the later developments of the work the adoption here of the Berlin magnitudes is to be regretted. It is unlikely, however, that its effect is sensible.

The stars were accordingly divided into four groups, whose limiting magnitudes are shown at the head of the columns of the following table. The means and weights are given as before for every three hours of R.A.

TABLE II.

R.A.	I. −6·9. Mean.	Weight.	II. 7·0−8·0 Mean.	Weight.	III. 8·1−8·9 Mean.	Weight.	IV. 9·0− Mean.	Weight.
h h	s		s		s		s	
0– 2	−·014	30	+·010	46	+·071	168	+·082	95
3– 5	+·026	27	+·048	25	+·079	218	+·106	184
6– 8	+·001	34	−·001	61	+·065	261	+·062	126
9–11	+·040	10	+·015	40	+·050	94	+·099	114
12–14	+·015	31	+·010	60	+·057	78	+·132	92
15–17	−·032	20	+·025	33	+·038	143	+·084	96
18–20	−·047	12	+·002	84	+·030	290	+·079	135
21–23	+·014	42	+·039	50	+·054	260	+·100	94
Totals...	+·003	206	+·018	399	+·056	1512	+·093	936

There can be no doubt that we have here arrived at the principal cause of the discrepancy between the Cambridge and Berlin Catalogues. The observations are affected by a personal equation depending on magnitude. It remains to determine whether the error is in one or both series.

Immediately after the conclusion of the zone observations for his catalogue, Dr. Becker made a series of experiments on thirteen evenings to evaluate the influence of magnitude on his R.A. observations. By using movable screens of fine wire gauze placed in front of the object-glass he was able to observe at the same transit stars alternately at their natural brightness and with their light diminished by a known amount. He came to the conclusion that, assuming the mean magnitude of the clock stars is $4^{m}\cdot0$, the R.A.'s of the Berlin Catalogue require a correction of $-{}^{s}\cdot007$ per magnitude – that is, that the fainter stars were observed late. Combining this result with those in Table II. we conclude that in both cases the R.A. observations are affected by a personal equation depending on magnitude, but that the amount of the equation is considerably greater in the Cambridge observations.

The work had just arrived at this stage when I received from Professor Turner, in connection with quite another matter, a table of the differences between the Oxford photographic measures and the Cambridge Catalogue places of twenty-six stars on one of the Astrographic Catalogue plates. It immediately occurred to me that the photographic results afforded the best possible test of the reality of the magnitude equation. For the photographic places are deduced in the following way:—From the assumed coordinates of the centre of the plate and the Cambridge Catalogue places the rectangular coordinates of all the Cambridge stars on the plate are computed, and compared with the actual measures on the plate of those stars. From the differences between the two the constants of the linear expression which corrects for error of centring and scale value, refraction, aberration, &c., are deduced, and the measured coordinates are corrected appropriately. The differences between the computed and the corrected observed coordinates of the known stars should then be merely accidental. It is evident, however, that if this personal equation depending on magnitude exists, the R.A. coordinate of the centre of the plate will be too large by an amount depending on the mean magnitude of the stars from which it is deduced. Hence all the x coordinates, corresponding practically to R.A.'s, which are deduced from the plate will be in error by the amount of the mean personal equation affecting the known stars. But the R.A.'s of brighter stars of the Cambridge Catalogue will be in error by less than this amount, and the fainter stars will be in error by more. When, therefore, we compare the measured and computed coordinates of the stars on the plate we shall expect to find the differences in the x coordinates (Oxford Photographic

—Cambridge Zone Catalogue) positive for stars brighter than the mean, negative for stars fainter.

I grouped the residuals in x coordinates from Prof. Turner's list according to the respective magnitudes of the stars, and it was immediately evident that those for the fainter stars were in general negative. This result was sent to Prof. Turner with a request for more material, and he very kindly placed at my disposal all he had then available, the residuals from seven plates. These were divided according to the magnitudes of the stars into four groups, which correspond exactly to the four groups in Table II., except that the magnitudes are Cambridge instead of Berlin. The quantities are decimals of a réseau interval, five minutes of arc. To reduce them to seconds of R.A. they must be multiplied by 20 sec $\delta = 22°3$.

TABLE III.

Mean x residuals from seven plates taken at Oxford for the Astrographic Catalogue.

No. of plate.	Coords. of R.A. h m	Centre. Decl. °	Mean mag. of known stars.	Mean residuals and number of stars from which they were deduced. I. —6·9		II. 7·0—8·0		III. 8·1—8·9		IV. 9·0—	
859	0 0	+26	8·9	+·0116	2	+·0063	2	—·0014	4	—·0014	20
888	0 40	+26	8·8	...		+·0019	3	+·0031	8	—·0014	18
929	0 48	+26	8·9	+·0071	1	+·0020	3	+·0039	5	—·0013	26
1045	0 48	+26	8·9	+·0085	1	+·0029	3	+·0047	5	—·0013	26
1046	0 56	+26	8·7	...		—·0004	3	+·0019	9	—·0067	11
866	2 0	+26	8·0	+·0004	4	+·0012	5	—·0044	7	+·0003	5
1049	2 32	+26	8·7	+·0024	1	+·0010	2	+·0010	13	—·0014	12

In general the number of stars on a plate which belong to any one group is so small that it is unsafe to base any conclusions on the value of the mean residual for a particular plate and group. In the reduction all the star places of the Cambridge Catalogue were given equal weight, though of many of the fainter stars—stars which were not included in the original programme of the Astronomische Gesellschaft—there was but a single observation. In these cases the error of the catalogue place may be comparatively large, and the corresponding large residual will in a small group entirely swamp the rest.

The apparently abnormal value for the third group of plate 866 seems, however, to be real. The seven residuals from which the given mean is derived are: $-^{R}\cdot0071$, $-^{R}\cdot0029$, $^{R}-\cdot0057$, $-^{R}\cdot0074$, $-^{R}\cdot0067$, $-^{R}\cdot0002$, $-^{R}\cdot0006$. This plate is, however, exceptional, in that the known stars on it are bright, their mean magnitude being 8·0. The residuals for the group of stars whose magnitudes are 8·1 – 8·9 would therefore be much smaller algebraically than usual, and it is evident that we cannot compare the means as they stand on this plate with those on the

other plates, upon which the known stars are much fainter. The value $+^{R}\cdot 0003$ for the last group on this plate is dominated by one large positive residual. Three of the four others are negative.

We have, then, in Table III. for any one magnitude group a number of mean residuals which differ greatly among themselves. A large part of this irregularity is due to accidental errors, since there are on a single plate but few stars belonging to the brighter groups. It is desirable, therefore, to combine the results of a large number of plates in order to get rid of this accidental error. The preceding paragraph, however, shows that it is not legitimate to take the simple mean of a column of the results in Table III., owing to the variation in the mean magnitude of the known stars on different plates. The question then arises, How can these results be combined?

The only systematic error which was to be feared in the photographic measures themselves was one due to personality in bisection. This has been entirely avoided at Oxford by turning the plates through 180° and re-measuring. The adopted measures, which are the means of the two series, will then be free from any such error. These mean measures in each coordinate are corrected by a linear formula, of which the constant term corrects for error of centring of the plate; and into the determination of this constant enter all the systematic errors of the catalogue from which the places of the known stars are taken.

The right ascension of a star in the Cambridge Catalogue may be considered sensibly free from personal equations depending upon velocity and direction of motion in the field, or upon the position of the instrument, since the stars all lie in a zone 5° broad, and the instrument was never reversed. It will involve the difference between the absolute personal equations affecting the observation of the star in question and the mean of those affecting the clock stars taken within an hour or two of it. It will also involve the mean error of the adopted R.A.'s of the clock stars, depending upon personal equations, error of adopted mean equinox, &c.

Let, then, E be the correction to be applied to the observed R.A. of a zone star, owing to absolute personal equation.

E_c the correction for a clock star.

F the mean error of the adopted R.A.'s of the clock stars.

Then for any star in the Cambridge Catalogue

Absolute R.A. = Cambridge R.A. + (E − mean E_c) + F.

And for the same star on the Oxford plates

$$\text{Absolute R.A.} = \text{Oxford R.A.} + \frac{1}{n}\Sigma\{\text{E} - \text{mean } E_c + \text{F}\}$$

if there are n known stars on the plate.

In general the same clock stars would be used for the whole of the region, 2° square, covered by one plate. Consequently the mean E_c is the same for all stars on the plate, and we have

The residual in the x coordinate, which is practically equal to Oxford R.A.$-$Cambridge R.A. $= E - \frac{1}{n}\Sigma(E)$.

Now this term $\frac{1}{n}\Sigma(E)$ is the mean of the absolute personal equations for all the stars on the plate, and will vary therefore for different plates. Even if the mean magnitude of the stars on two plates is the same, the value of $\frac{1}{n}\Sigma(E)$ will not in general be the same, unless E can be expressed as a linear function of the magnitude. But it is in any case a constant for the plate $=$ C suppose. If, therefore, the personal equations for two stars are E and E′, and the corresponding residuals R and R′, we have

$$E = R + C$$
$$E' = R' + C,$$

and

$$E - E' = R - R'.$$

We can therefore combine the *differences* between the corresponding quantities in the columns of Table III. without any reference to the magnitudes of the known stars on the plates. This result is perhaps sufficiently obvious from the consideration—which is a summary of the above reasoning—that the individuality of the plate, so far as personal equation is concerned, enters only into the constant which corrects for the error of centring of the plate, and cannot, therefore, affect the difference between two measures on the same plate; for except in the case of a very special distribution with respect to magnitude of the known stars on a plate, the error in the determination of the coefficients of x and y in the linear correction formula which will be produced by the adoption of the slightly erroneous values of the R.A.'s may be considered negligible. It is, however, not quite so obvious that the quantities obtained by subtracting the columns of Table III. are differences of absolute personal equation, and involve neither errors in the R.A.'s of the clock stars nor personal equation in observing them. It seemed worth while, therefore, to retain the detailed consideration of this point which is contained in the preceding paragraphs.

We have in Table III. practically results from only six plates, since 929 and 1045 are duplicates. The mean of the two has been treated as one plate. If, then, we form the differences between the columns of Table III., weight them by the formula used above, and take the means, we have—

Groups.	Mean Residuals. R	s	Weights.
IV.–I.	−·0061	= −·136	17
IV.–II.	−·0039	= −·087	37
IV.–III.	−·0032	= −·071	55

Comparing these with the results deduced from the comparison of the Berlin and Cambridge Catalogues we have—

TABLE IV.

	Comparison of Cambridge and Berlin A Uncorrected Berlin Personal Equation. s	B Corrected Equation. s	Oxford Photographic Measures C. s
Stars in Group IV. are observed later than stars in Group I. by	·090	·110	·136
later than stars in Group II. by	·075	·086	·087
and than stars in Group III. by	·037	·042	·071

The second column has been formed on the assumption that the Berlin R.A.'s must be decreased ^{s}·007 per magnitude (see above), and that the stars of Groups I., II., and III. are on an average $2^{m}\cdot8$, $1^{m}\cdot6$, and $0^{m}\cdot6$ respectively brighter than those of Group IV.

The general agreement of these two determinations in columns B and C of the effects of the personal equation must be considered satisfactory, when due allowance is made for the small amount of material upon which the photographic results are based. A considerable disagreement between columns A and C would be no argument against the validity of the photographic method, since the quantities in the third column are the differences of absolute personal equation between the different magnitude groups, and in the first column the differences of relative personal equation—that is, the differences of the values of the quantity "Cambridge–Berlin personal equation."

The third column, on the other hand, should agree with the second, since the Berlin results used in forming it have been corrected by the value of the personal equation, $-^{s}$·007 per magnitude, which was found at Berlin from observations made with wire gauze screens. This method is, however, open to the serious objection that the change in the character of the star's image due to the diffraction of the screen is very likely to have an appreciable influence on the observer's habit. The results of Table IV. seem to show, moreover, that the personal equation function is not linear, at least in the case of the Cambridge observations.

The method used above of dividing the stars into four groups according to their magnitudes was sufficiently exact to show the dependence of the personal equation upon magnitude. When, however, we come to the problem of determining the form of the function of the magnitude which represents the personal equation this method is insufficient, and at present there is not enough material available for an exact investigation. But there is evidence besides that of Table IV. which seems to show that the function is not linear.

It was noticed during the grouping of the residuals for the

formation of Table III. that of those that went into the last group, magnitude 9·0 and fainter, the residuals belonging to the stars marked 9·0 were in general positive, and that the mean of the group was rendered negative only by the somewhat large negative residuals for the fainter stars. They were therefore collected for each tenth of a magnitude from $8^m{\cdot}5$ downwards, neglecting the results from plate 866. The following table gives the number of stars in each group and the mean residual :—

TABLE V.

Mag.	No. of Stars.	Mean Residual.	Mag.	No. of Stars.	Mean Residual.
8·5	6	R + ·0033	9·1	11	R − ·0020
8·6	6	+ ·0016	9·2	13	− ·0105
8·7	5	+ ·0010	9·3	13	− ·0052
8·8	3	− ·0028	9·4	6	− ·0057
8·9	6	+ ·0011	9·5	18	− ·0026
9·0	24	+ ·0049	...	...	...

It appears from this table that the assumed magnitudes of the stars are not very well distributed. Apparently there has been a tendency to mark stars $9^m{\cdot}0$ rather than $8^m{\cdot}7$, $8^m{\cdot}8$, or $8^m{\cdot}9$. This tendency would, however, have made somewhat too large the assumed mean magnitude, $8^m{\cdot}8$, of the stars on the plates under discussion at the same time that it increased the number of stars marked $9^m{\cdot}0$ having a positive residual. It can hardly, then, affect the important indication which this table affords, that the value of the mean residual vanishes, not for stars of the mean magnitude, but for stars sensibly fainter. The meaning of this result is sufficiently obvious—the function of the magnitude which represents the personal equation for that magnitude is not linear. If a curve were drawn having magnitudes as abscissæ, and the corresponding personal equations as ordinates, it would become much steeper for the fainter stars.

It has been convenient so far to consider the personal equation a function of the magnitude—that is, of the logarithm of the brightness of a star. It is quite probable, however, that it can be better represented as an algebraical function of the brightness. The point cannot be settled until there is more material available for discussion.

To sum up :—There is a small systematic difference between the R.A.'s of the Cambridge and Berlin Zone Catalogues, which varies with magnitude, and is probably due to a personal equation depending on magnitude. If we accept as final the results obtained at Berlin with wire gauze screens, faint stars were observed late at both observatories, but the effect was rather more pronounced at Cambridge. This is confirmed on the whole by a discussion of residuals which occur in the reduction of a few of the Oxford Astrographic Catalogue plates, and for a

complete investigation of the question these residuals form the best possible material. When, therefore, the reductions for the Astrographic Catalogue are completed it will be possible to discuss very accurately the personal equations depending on magnitude in the observations for the meridian catalogues upon which it is based.

In conclusion, I must express my thanks to Professor Turner for the results from the Oxford University Observatory which he has kindly placed at my disposal, and I am also indebted to him and to Mr. Newall for much very helpful advice and criticism.

The Observatory, Cambridge:
1897 *April* 8.

An Investigation concerning the Position Error affecting Eye-Estimates of Star Magnitudes. By Alexander W. Roberts.

Measures of star magnitudes according to the method of Argelander, or any of its modifications, are always liable to be seriously vitiated by several disturbing influences. Attention has been directed from time to time to these sources of error by astronomers who have made special study in this branch of astronomical research ; not infrequently they have been pointed out in order to throw discredit on the determination of star magnitudes by eye-estimates.

It is evident, however, that the mere existence of error in any series of measurements, of whatever nature, cannot condemn the measurements made, unless it is also certain that the errors cannot be avoided or eliminated. Now with ordinary precautions the chief errors that influence eye-estimates of stellar brightness can be entirely negatived, either by avoiding them or by adopting expedients which render them inoperative.

One of the principal influences modifying almost every observation made by eye-estimates is that arising from the ever-changing relative position of a variable and its surrounding comparison stars. Put generally, the error arises from the tendency, common, apparently, to all observers, to over-estimate the brightness of the lower star or stars in any field. If the amount of error were insignificant, it might be neglected without endangering seriously the substantial accuracy of any determination depending upon light measurements ; but the amplitude of error is frequently almost a whole magnitude, an amount much greater than the actual variation of some shorter-period variables. Then in addition to the magnitude of error arising from this source we have its systematic nature. Now it is evident that a subjective systematic variation in the magnitude of any star whose light changes are under consideration will find expression

in definite terms, the derivation of which it will be impossible to assign either to subjective change due to unequal sensitiveness of different parts of the retina, or to actual objective changes in the light of the star itself.

Thus we may have an apparent certain variation in the light of a star that after all is but the result of this tendency to over-estimate the brightness of the lower of two objects. It is pertinent, therefore, to the subject of stellar astronomy to investigate as thoroughly as possible a phenomenon that bears so directly on the accurate determination of star magnitudes.

The error may be avoided by taking observations when the star is in the same hour angle. This is not, however, always possible, and with rapidly varying stars it is out of the question. Then, again, the error may be avoided by the observer keeping his head always in the same position with relation to the stars under examination. For very evident reasons this expedient has been found to be practically unworkable. Another method of overcoming the difficulty will naturally suggest itself to many. The field under examination might always be kept in the same position with respect to the vertical by means of a rotating prism. In this case "position error" (using this term to describe the error now under consideration) would affect the absolute magnitudes of each star, but the excess or defect being always a constant quantity, the variation of the star would in no way be influenced.

None of the foregoing methods yield any data for an investigation as to the amount and nature of "position error." They are, indeed, effective expedients for its elimination, but they afford no means of ascertaining the amount of error eliminated.

I have had it in purpose for some years to adopt only such a mode of observation as would yield simply and readily the amount of error due to subjective causes, and the method that seemed best suited for such a purpose was that in which two observations are always taken—one direct, and the other reverse. This is secured by using for each observation two eye-pieces—one astronomical, and the other terrestrial.

It is evident that whatever stars are over-estimated when using one eye-piece, will be under-estimated to an equal extent when the other eye-piece is used. Thus a mean of the two observations will yield a set of measures entirely freed from position error. But, further, the difference between the two sets of measures (when properly reduced to a common standard) will yield *twice the amount of position error affecting any one of the stars at the time of observation.* Then if complete observations of the same set of stars be made in at least twelve hour angles, we shall have all the data necessary for a full investigation of the relation existing between the varying magnitudes and the changes in position producing the variation.

Instead of utilising all available observations I think it will give definiteness to the investigation to limit the area under

examination to one group of stars. For such an investigation several groups of stars are available, but I think the group that yields itself most readily to the investigation is that surrounding the short period variable, Lac. 5861.

The positions of this variable and its surrounding comparison stars are as follows :—

A.Z.C.	Lac.	R.A. (1875).	Dec.	Mag.
		h m s	° ′ ″	
259	5834	14 3 57·6	−57 43 39	7·61
505	5859	7 48·1	56 42 45	7·26
512	...	7 51·1	57 44 29	7·87
533	5861	8 10·2	57 16 12	Var.
639	5870	9 56·8	58 6 21	7·40
837	5888	14 13 8·0	57 42 58	7·56

It will be evident from the accompanying chart that the configuration of the group (fig. 1) is in every way advantageous

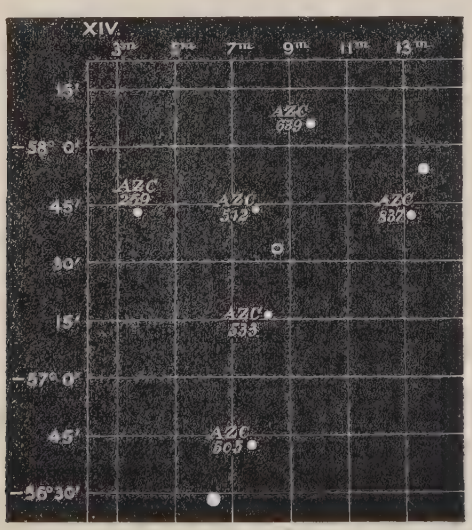


Fig. 1.

to a full investigation. Attention at this point may be called to the position marked O in the chart; this is the centre of the field, and is found in the usual way by triangulating the stars in the system.

The coordinates of O are :—

R.A. $14^h\ 8^m{\cdot}5$

Dec. $-57°\ 33'{\cdot}0$.

The line passing from the zenith through this point will be called the vertical line.

The mode of reducing the estimated magnitudes to standard values bears directly on the question under consideration, and may briefly be described here.

Let

$$M_{(1)} + M_{(2)} + M_{(3)} + M_{(4)} + M_{(5)} = N$$

be the magnitudes of the five comparison stars A.Z.C. 259, 505, 512, 639, 837, as determined by sequences from $6^m{\cdot}8$ at a single observation, N being the sum total of the five magnitudes.

Let M_v be the magnitude of the variable A.Z.C. 533 (Lac. 5861) as determined with relation to the five comparison stars.

From all the observations a mean value of N is found: the average of over 500 values of N gave as this mean value

$$37{\cdot}70.$$

Then to reduce the magnitudes of the five comparison stars to mean standard value, the correction

$$\frac{37{\cdot}70 - N}{5}$$

is applied to each star, making the sum total of the five magnitudes,

$$M_{(1)}, M_{(2)}, M_3, M_{(4)}, M_{(5)},$$

always equal to 37·70.

The same correction is also applied to the variable, which therefore becomes

$$M_v + \frac{37{\cdot}70 - N}{5}.$$

This reduction is rigorously carried out for all observations direct and reverse.

The reduction being completed, the difference between the reverse and direct observations will represent twice the error due to the relative positions of the six stars in the group, the variable and the five comparison stars.

In the accompanying table the differences are taken in groups of one hour, and the parallactic angle of O is given in the same table, as the reduction of the differences will naturally be related in the first instance to this angle.

Table I.

Rot. No.	No. of Obs.	Sidereal Time.	Parallactic Angle.*	Reverse—Direct Observations. A.Z.C. 259.	A.Z.C. 505.	A.Z.C. 512.	A.Z.C. 593.	A.Z.C. 639.	A.Z.C. 837.
		h m	° ′	m					
1	4	5 46	44 12	−0·54	+0·54	−0·14	+0·16	−0·44	+0·46
2	4	6 37	54 0	·62	·53	·19	·18	·47	·55
3	5	7 29	63 50	·57	·53	·21	·13	·47	·57
4	16	9 40	89 39	·79	·44	·18	·13	·21	·62
5	26	10 27	100 11	·82	·30	·17	+0·09	−0·05	·65
6	28	11 27	115 46	·83	+0·14	·11	−0·02	+0·04	·77
7	35	12 32	137 52	·71	−0·32	−0·02	·09	·29	·85
8	49	13 29	161 1	·46	·57	+0·03	·22	·57	·65
9	61	14 27	188 42	−0·19	·65	·03	·32	·73	·38
10	55	15 29	216 14	+0·08	·68	·04	·33	·78	+0·11
11	32	16 29	237 39	·39	·73	·10	·23	·68	−0·20
12	37	17 33	255 27	·61	·63	·13	·11	·51	·51
13	40	18 29	268 24	·73	·54	·20	·10	·34	·65
14	42	19 26	280 10	·80	·34	·18	·07	+0·16	·74
15	38	20 24	291 26	·87	−0·06	·13	·04	−0·05	·85
16	20	21 27	303 20	·78	+0·20	·05	−0·00	·11	·93
17	8	22 32	315 48	+0·67	+0·49	+0·02	+0·10	−0·32	−0·93

If the residuals in the preceding table be plotted down, and a curve drawn through the positions, it will be found that all the five curves can be well represented by an expression of the form,

$$x + \alpha \cos (q - M) + \beta \cos (2q - N).$$

From the nature of the error under investigation the first term of the above expression should be sufficient to satisfy the residuals of Table I., but as it is extremely improbable that observations 180° of parallactic angle, apart, will be taken under the same conditions, the second term is retained.

A rigorous least squares solution of the seventeen sets of observations given in Table I. (weighting each set according to the number of observations in the set) yielded the following numerical values for the variation of the position error, depending on q, the parallactic angle at O.

(1) A.Z.C. 259.

$$-0^m{\cdot}04 + 0^m\,82 \cos (q - 288°\ 22') + 0^m{\cdot}08 \cos (2q - 270°).$$

* The *parallactic angle* is here considered to be the angle formed at O by the lines passing through the zenith and the south pole.

(2) A.Z.C. 505.

$$+0^{m}\cdot04+0^{m}\,85\cos(q-27^{\circ}\,34')+0^{m}\cdot08\cos(2\,q-357^{\circ}).$$

(3) A.Z.C. 512.

$$-0^{m}\cdot04+0^{m}\cdot18\cos(q-240^{\circ}\,50')+0^{m}\cdot06\cos(2\,q-336^{\circ}).$$

(4) A.Z.C. 533.

$$-0^{m}\cdot05+0^{m}\cdot22\cos(q-28^{\circ}\,40')+0^{m}\cdot06\cos(2\,q-198^{\circ}).$$

(5) A.Z.C. 639.

$$+0^{m}\cdot05+0^{m}\cdot70\cos(q-200^{\circ}\,50')+0^{m}\cdot07\cos(2\,q-97^{\circ}).$$

(6) A.Z.C. 837.

$$+0^{m}\cdot05+0^{m}\cdot82\cos(q-121^{\circ}\,47')+0^{m}\cdot09\cos(2\,q-72^{\circ}).$$

A comparison between the observed values given in Table I. and the values computed from the preceding expressions is given in the following table.

Rot. No.	A.Z.C. 259. Obs	Comp.	O−C.	A.Z.C. 505. Obs.	Comp.	O−C.	A.Z.C. 512. Obs.	Comp.	O−C.	A.Z.C. 533. Obs.	Comp.	O−C.	A.Z.C. 639. Obs.	Comp.	O−C.	A.Z.C. 837. Obs.	Comp.	O−C.	Rot. No.
	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	
1	−0·54	−0·49	−0·05	+0·54	+0·85	−0·31	−0·14	−0·26	+0·12	+0·16	+0·15	+0·01	−0·44	−0·52	+0·08	+0·46	+0·31	+0·15	1
2	·62	·60	−0·02	·53	·77	−0·24	·19	·25	+0·06	·18	·16	+0·02	·47	·47	+0·00	·55	·44	+0·11	2
3	·57	·70	+0·13	·53	·67	−0·14	·21	·24	+0·03	·13	·15	−0·02	·47	·40	−0·07	·57	·54	+0·03	3
4	·79	·82	+0·03	·44	·43	+0·01	·18	·18	+0·00	·13	·12	+0·01	·21	·25	+0·04	·62	·72	−0·10	4
5	·82	·82	+0·00	·30	+0·22	+0·08	·17	·14	−0·03	+0·09	·08	+0·01	−0·05	−0·09	+0·04	·65	·74	−0·09	5
6	·83	·79	−0·04	+0·14	−0·03	+0·17	·11	·09	−0·02	−0·02	+0·02	−0·04	+0·04	+0·06	−0·02	·77	·76	+0·01	6
7	·71	·67	−0·04	−0·32	·24	−0·08	−0·02	−0·03	+0·01	·09	−0·11	+0·02	·29	·30	−0·01	·85	·74	+0·11	7
8	·46	·49	+0·03	·57	·47	−0·10	+0·03	+0·00	+0·03	·22	·23	+0·01	·57	·54	+0·03	·65	·65	+0·00	8
9	−0·19	−0·21	+0·02	·65	·68	+0·03	·03	·03	+0·00	·32	·32	+0·00	·73	·75	−0·02	·38	·43	−0·05	9
10	+0·08	+0·12	−0·04	·68	·77	+0·09	·04	·07	−0·03	·33	·30	−0·03	·78	·79	−0·01	+0·11	+0·01	+0·10	10
11	·39	·39	+0·00	·73	·72	−0·01	·10	·11	−0·01	·23	·24	+0·01	·68	·68	+0·00	−0·20	−0·23	+0·03	11
12	·61	·60	+0·01	·63	·59	−0·04	·13	·14	−0·01	·11	·16	+0·05	·51	·49	+0·02	·51	·50	−0·01	12
13	·73	·72	+0·01	·54	·45	−0·09	·20	·15	+0·05	·10	·10	+0·00	·34	·33	+0·01	·65	·67	+0·02	13
14	·80	·79	+0·01	·34	·28	−0·04	·18	·14	+0·04	·07	·06	−0·01	+0·16	+0·16	+0·00	·74	·77	+0·03	14
15	·87	·83	+0·04	−0·06	−0·10	+0·04	·13	·13	+0·00	·04	−0·02	−0·02	−0·05	−0·10	+0·05	·85	·84	−0·01	15
16	·78	·83	−0·05	+0·20	+0·10	+0·10	·05	·10	−0·05	−0·00	+0·01	−0·01	·11	·26	+0·15	·93	·87	−0·06	16
17	+0·67	+0·76	−0·09	+0·49	+0·31	+0·18	+0·02	+0·05	−0·03	+0·10	+0·04	+0·06	−0·32	−0·41	+0·09	−0·93	−0·84	−0·09	17

So far we have considered the values of the errors, the terms expressing their amount and variation, as depending upon the parallactic angle of point O, the central point of the group of six stars. We now arrive at an important transference of argument.

The line passing from the zenith through O we have agreed to call the vertical line. Let, for brevity, the angle formed at O by this vertical line and a line joining O with any one of the five stars be called the position angle. Does the position error reach a maximum (or a minimum) for all stars in the group now under consideration at the same position angle?

It is possible to solve this question graphically, but in a first investigation it is preferable to proceed otherwise.

Let O be the origin of coordinates, and let O_1 be another point to which the measurements of position angle and parallactic angle are referred. Then taking

α and β coordinates of O_1 ;
α_1 and β_1 coordinates of star ;
κ parallactic angle at O when position error is at a maximum ;
θ constant position angle when position error is at a maximum ;

each maximum of position error (obtained from Table I.) will yield equations of the form

$$(\beta_1 - \kappa\alpha_1)\,\theta + \kappa\,(\theta\alpha + \beta) + (\alpha - \theta\beta) = (\kappa\beta_1 + \alpha_1).$$

It is unnecessary to burden the present paper with the numerical steps leading to the following results :

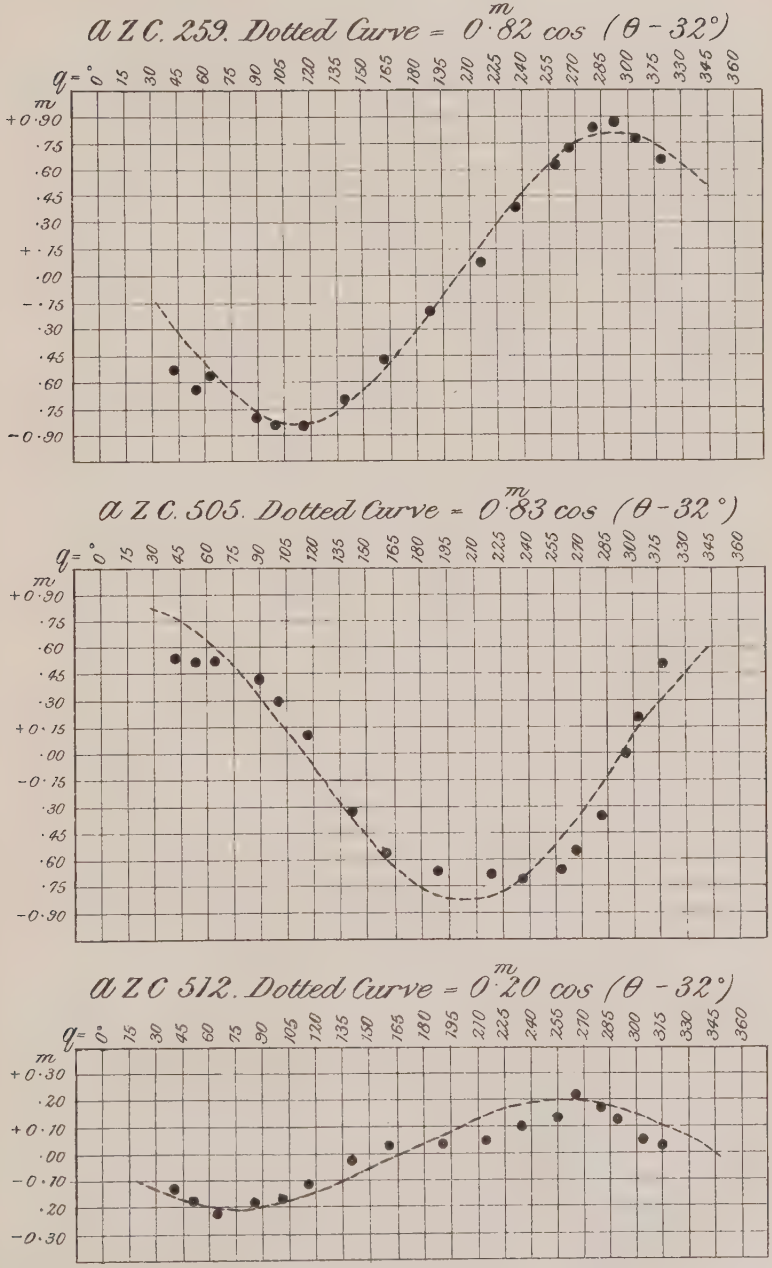
$$\alpha = +1''{\cdot}5$$
$$\beta = -4''{\cdot}3$$
$$\theta = \quad 32^\circ.$$

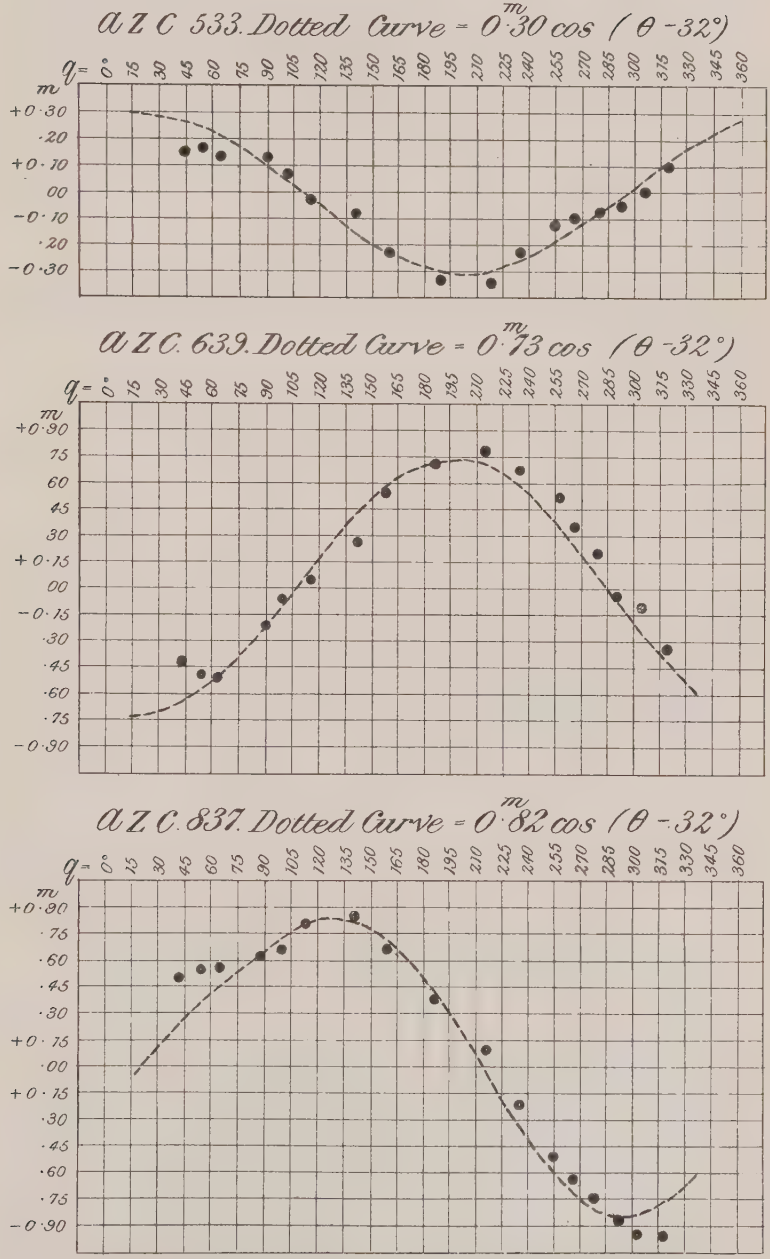
That the equation should yield as a mean point of reference a position only *four and a half minutes of arc* distant from the central point of the field is very remarkable when we consider the character of the investigation. Theoretically I am convinced that the two points O and O_1 coincide, but uncorrected sources of error—as, for example, a slight change in the position of the head when eye-pieces of different lengths are used—combine to produce the slight non-agreement of the two positions.

Taking, therefore, as our mean point of reference a position

	h	m
R.A.	14	8·7
Dec.	− 57°	37·3′,

we find that the position errors affecting the magnitudes of the six stars under consideration reach their maximum when the position angle at O is 32°. Or, to put this statement more generally, with


α Z C. 259. Dotted Curve = 0ᵐ·82 cos (θ − 32°)
q = ° 0 15 30 45 60 75 90 105 120 135 150 165 180 195 210 225 240 255 270 285 300 315 330 345 360
m
+0·90 ·75 ·60 ·45 ·30 +·15 ·00 −·15 ·30 ·45 ·60 ·75 −0·90
α Z C. 505. Dotted Curve = 0ᵐ·83 cos (θ − 32°)
q = ° 0 15 30 45 60 75 90 105 120 135 150 165 180 195 210 225 240 255 270 285 300 315 330 345 360
m
+0·90 ·75 ·60 ·45 ·30 +0·15 ·00 −0·15 ·30 ·45 ·60 ·75 −0·90
α Z C 512. Dotted Curve = 0ᵐ·20 cos (θ − 32°)
q = ° 0 15 30 45 60 75 90 105 120 135 150 165 180 195 210 225 240 255 270 285 300 315 330 345 360
m
+0·30 ·20 +0·10 ·00 −0·10 ·20 −0·30



Note: θ = Angle formed at centre of field by the star & the nadir.

LEEDS PHILOS.
AND LITERARY

any system of stars whose magnitudes are determined visually each star will reach an apparent maximum brightness when the angle subtended by a line drawn from the star to the centre of the field and the vertical line at this point is 32°.

How close a general expression for all the six stars meets the observed differences between reverse and direct observation may be evidenced by graphical illustrations.

The dotted curves (Plates 12 and 13) are computed from the general formula

$$r \cos (\theta - 32°).$$

I am unable to obtain a general expression for r, but it would seem that the further from the centre O the star is, the greater will the coefficient r be. The following table gives the relation :

	Star.	Distance of Star from O.	Observed Values of r.
		′	m
1	A.Z.C. 505	55·1	0 83
2	259	38·3	0 82
3	837	36 6	0·82
4	639	30·8	0 73
5	533	21·5	0·30
6	512	9·6	0 20

The disturbing effects of position error have been commented upon by Safarik (*Ast. Nach.* Band 101, page 24).

It will be seen that the relation existing between the distance of a star from the centre of the field and the amount of error which influences comparative eye-estimates of its brightness is by no means an intimate one ; indeed, we can readily understand circumstances—as, for example, twenty stars in a line—where the relation would naturally break down.

The foregoing investigation has been confined to the group of stars surrounding L. 5861 ; but whatever group be considered, as far as Lovedale observations are concerned, the general results are the same.

(1) The error due to relative position is systematic in its nature.

(2) It reaches its maximum effect when the position angle of the star with relation to the vertical at the centre of the field is 32°.

(3) It affects all magnitudes equally.

These results are absolute as far as my personality is concerned ; whether they hold good for other observers I cannot say. It must not be forgotten, however, that the error has been commented upon at some length by Safarik (*Ast. Nach.* Band 101, p. 24); by Pickering (*Harvard Annals*, vol. xiv. part 1, p. 42); and by Chandler (*Astronomical Journal*, No. 374, p. 107). The two latter astronomers have also proved in the papers referred to the systematic nature of the error. I may be permitted also to say

that Dr. Gould, when corresponding with me some years ago on the matter, informed me that the phenomenon was frequently an anxiety to the Cordoba observers. It is not unreasonable, therefore, to conclude that the error, in at least its general characteristics, influences all eye-estimates of star magnitudes by the method sequences. That there are possibilities, therefore, of grave misleading in discussing observations, in the securing of which no steps have been taken, either through want of care or want of knowledge, to eliminate so important an error, is surely beyond reasonable doubt. Indeed, it is certain that every result based on uncorrected observations is more or less in error.

Whether some of the secondary terms that appear so often in the computed elements of variable stars have their origin in this source of error I cannot say ; only it is possible for position error to give rise to such secondary terms when left uncorrected. It is unnecessary to dwell upon the connection existing between this tendency and the exceedingly long lists of suspected variables which now and again find a lodging place in astronomical literature.

The assumption all through this paper, as well as in the papers of Safarik, Pickering, Chandler, and Barnard, is that the error has its source in the eye of the observer. I have convinced myself that this is so by a very simple expedient. The telescope I use is a small 1-inch theodolite. To make sure that the error was not due to the nature or adjustments of the instrument, I rotated the small telescope in its Ys, thus taking the observations with the object glass in all positions. I also used a variety of eye-pieces. Further, when the head was rotated through a certain angle, the same changes in the relative magnitudes of the stars seemed to take place as would have been observed had the stars moved through an equal hour angle.

The error also operates in cases when no optical aid is required.

Proper Motion of the Southern Short Period Variables L Carinæ and K Pavonis. By Alexander W. Roberts.

The following proper motions of *L Carinæ* and *K Pavonis* have been determined solely from observations made at the Cape Observatory. The measures available are the positions given in the General Catalogues for 1840, 1850, 1860, 1880, and the meridian results of 1887, 1888, 1889, 1890, 1891.

The mean places are reduced to 1900·0, using Newcomb's precession constants (*Astron. Constants*, p. 196), and the mean latitude of the Cape Observatory is held to be

$$-33^\circ\ 56'\ 3''{\cdot}52.$$

Whenever possible Chandler's corrections for latitude variation have been applied, and the various catalogues have been differentially compared.

L Carinæ.

No.	Epoch.	Taken from.	R.A.	Red.	R.A. (1900).	Dec.	Red.	Dec. (1900).	No. of Obs.	Weight
			h m s	m s		° ′ ″	′ ″			
1	1840	Cat. 1840	9 40 51·16	+1 38·99	9 42 30·15	−61 46 18·11	−16 30·60	−62 2 48·71	6–9	2
2	1850	Cat. 1850	41 7·74	1 22·48	30·22	49 2·49	13 46·05	48·54	1–3	1
3	1860	Cat. 1860	41 23·96	1 5·98	29·94	51 46·82	11 0·90	47 72	4–4	1
4	1880	Cat. 1880	41 57·07	32·99	30·06	57 17·46	5 30·65	48·11	3–3	1
5	1890	Mer. Result, 1887–91	42 13·34	16·49	29·83	60 2·42	2 45 85	48·27	12–12	3

A discussion of the above reduced places yields the following :—

R.A. (1900) $9^h\ 42^m\ 29^s\cdot795$. P.M. $= -0^s\cdot006$. Dec. $-62°\ 2'\ 48''\cdot09$. P.M. $= +0''\cdot007$.

K Pavonis.

No.	Epoch.	Taken from.	R.A.	Red.	R.A. (1900).	Dec.	Red.	Dec. (1900).	No. of Obs.	Weight.
			h m s	m s		° ′ ″	′ ″			
1	1840	Cat. 1840	18 40 25·12	+6 13·34	18 46 38·46	...	...	...	5–0	2
2	1850	Cat. 1850	41 27·59	5 11·02	38·61	−67 24 42·58	+ 3 11·07	−67 21 31·51	2–2	1
3	1860	Cat. 1860	42 29·59	4 8 73	38·32	24 7·03	2 35·19	31·84	3–3	1
4	1880	Cat. 1880	44 34·09	2 4·27	38·35	22 50·75	1 18·73	32·02	3–3	1
5	1890	Mer. Result, 1887–91	45 36·22	1 2·11	38·33	22 11 99	39·88	32·11	10–10	3

A discussion of the above places yields the following : —

R.A. (1900) $18^h\ 46^m\ 38^s\cdot303$. P.M. $-0^s\cdot003$. Dec. $-67°\ 21'\ 32''\cdot26$. P.M. $-0''\cdot013$.

An Improvement in Winding Equatorial Clocks.

By A. F. Lindemann.

All equatorial clocks are, as far as I am aware, disturbed during winding; but the disturbance is usually hidden by the momentum of the heavy governor. My intention being to detect and correct all driving errors, I constructed an exceedingly light governor, weighing about $\frac{1}{4}$ oz., and rotating 540 times a minute, the clock being corrected every second by a main pendulum clock. With this arrangement the winding error was most conspicuous, until the new winding was introduced, with the result that a star set to the vertical line of the micrometer and observed with a power of 214 showed absolutely no movement during the most irregular starting, winding, or stopping. The construction of new winding is as follows (Plates 14 and 15, figs. 1, 2 and 3):

I took barrel out of clock (an old one by Cook) and inserted in its place a double V-grooved cast iron sheave (1 and 2) and fixed it to the axle C by a steel screw and by a pin resting against one spoke of the main driving wheel D. Exactly under the axle C I placed the winding axle E, held in position on one side by a screw and washer, on the other by a ratchet-wheel R and ratchet r and a double V-grooved cast-iron sheave 1_1 and 2_1, of the same dimensions as 1 and 2, and carrying on the prolongation of E the winding crank d. Ratchet-wheel and sheave are soldered to E. Fixed to the iron clock support G is the fork H, which carries at the outer end the axle I, upon which the ⌣-grooved loose sheaves a b and a_1 b_1 are placed, secured on both ends by washers and screw heads. Under this fork and just in front of clock support G are fixed by means of a plate and axle the ⌣-grooved guiding rollers c and c_1. An endless clock string, with long splice of 3 feet and about $\frac{3}{16}$-inch thick, carries the weight P and counter weight p as seen in fig. 3, and starting say at β, passing on c and the front of 1 returns to top of a, crossing over to 2, whence it returns over top of b down to sheave r^1, carrying counter weight p, rises on the other side to sheave b_1, passing on and under 2_1, from there crossing over to and under a_1 and over to and under 1_1, from which it passes on to c_1, descending to sheave R (to which is attached the large driving-weight P), and returns from there to β. The action is obvious, the weight P is always with $\frac{P}{2} - \frac{p}{2}$ on driving wheel D. During winding the same is the case, and consequently there is no change in the clock's rate. When winding, all that happens is that the string with P is pulled up by sheave 1_1 and 2_1, the weight p descending the same amount. The ratchet prevents the turning back of axle E with its sheaves, and the whole winding is done by friction. I rejected the chain, which would have been simpler,

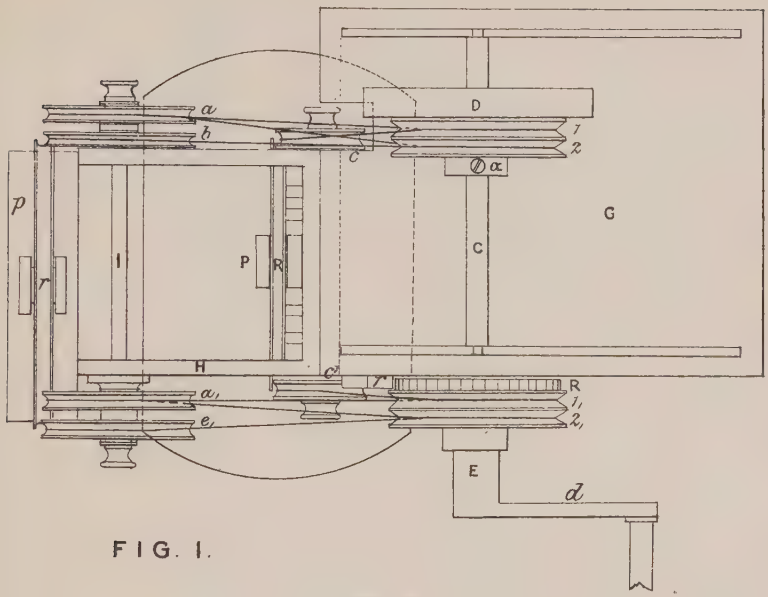

D
a
b
C
1
2
α
G
p
I
P
R
C
H
a,
e,
R
1,
2,
E
d

FIG. 1.

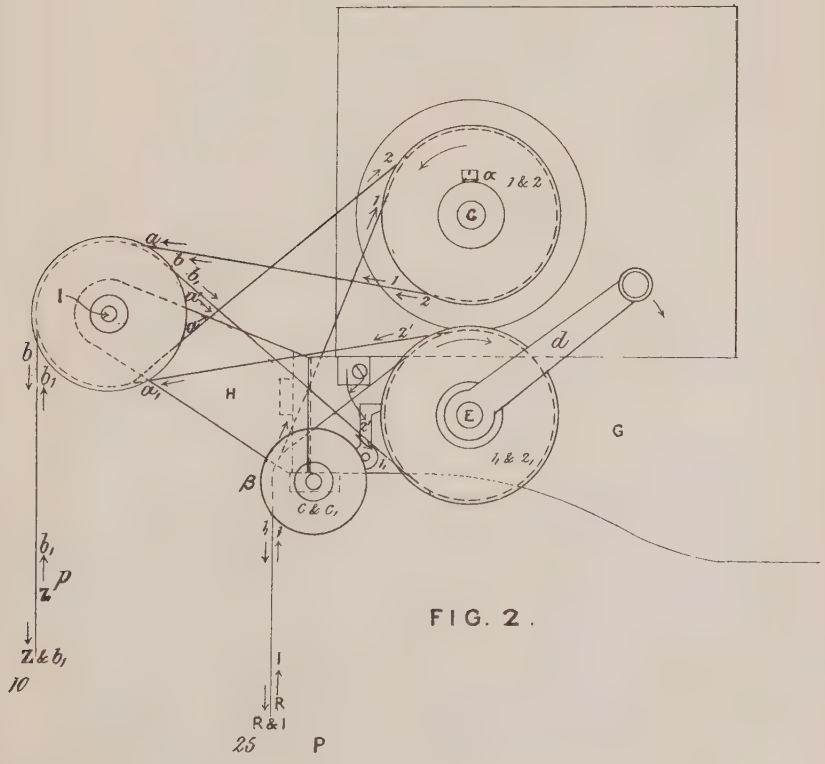
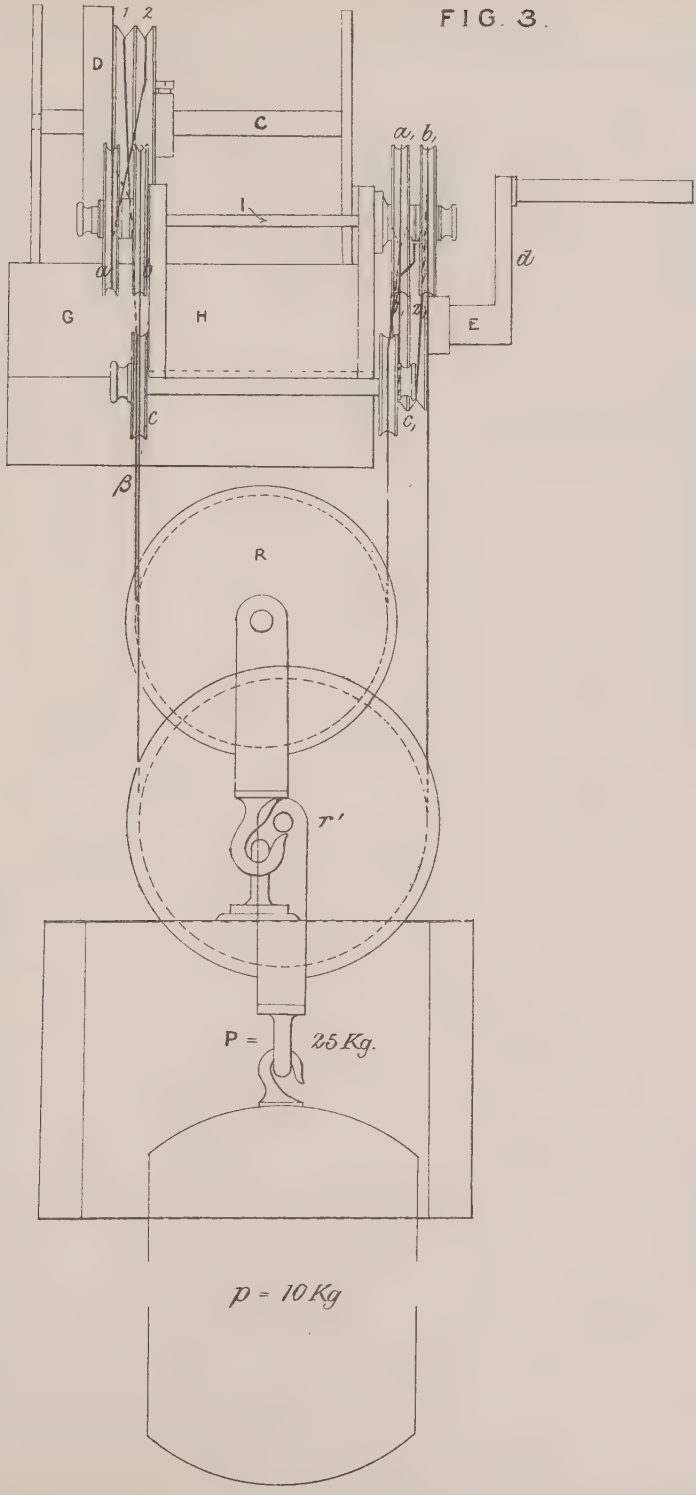

α 1 & 2
C
2
1
a
b
b,
a,
a'
a'
I
1
2
z'
d
H
G
E
1, & 2,
β
C & C,
b
b,
a,
1
1
b,
z
p
Z & b,
10
I
R
R & I
25
P

FIG. 2.

FIG. 3.



on account of the possibility of uneven working, and fearing that each link might show, which is here avoided. The position of the pulleys a b and a_1 b_1 have been so calculated that they give the greatest amount of frictional angle upon 1 and 2 and 1_1 and 2_1. The friction was primarily calculated and proved a perfect success. The weight p can be lifted without the string slipping. The position of the various sheaves to one another is also most important, the string having to pass itself several times on each side without touching. Cast-iron was chosen, having a great frictional resistance. Naturally in a new clock the arrangement could be made much more compact. This method of winding, the idea of which is old and well known, is only new in so far as it acts without a chain and entirely by friction.

This winding has been in constant use during the last eight months in my observatory with perfect success.

Sidmouth Observatory:
1897 *April* 8.

Observations of Minor Planet (8) *Flora at Windsor, N.S. Wales.* By John Tebbutt.

The accompanying observations of *Flora* have been made with the 8-inch equatorial and filar micrometer and in a bright field. The micrometer differences have been corrected for refraction and a small error in the perpendicularity of the micrometer threads. The quantities $p\Delta$ are based on an equatorial parallax of the sun=8″·85.

(8) *Flora.*

Date. 1896.		Windsor Mean Time. h m s	Planet−Star. Δ R.A. m s	Δ N.P.D. ′ ″	No. of Comps.	Planet's App. R.A. h m s	Log. $p\Delta$	Planet's App. N.P.D. ° ′ ″	Log. $p\Delta$.	Reductions to App. Place R.A. s	N.P.D. ″	Comp. Star.
Dec.	1	10 53 10	+2 56·46	+ 7 19·7	12	4 41 52·98	9·154*n*	75 41 2·2	0·811	+5·17	−16·8	1
	1	11 4 19	+1 6·86	+ 1 47·6	8	4 41 52·13	9·074*n*	75 41 1·3	0·812	+5·17	−16·6	2
	4	10 27 46	−0 24·15	+ 1 6·8	19	4 38 32·11	9·216*n*	75 34 49·5	0·810	+5·21	−16·6	1
	4	10 27 46	−2 13·17	...	19	4 38 32·14	9·216*n*	...	...	+5·21	...	2
	5	10 38 42	+0 14·01	+ 9 47·4	15	4 37 24·85	9·115*n*	75 32 32·5	0·813	+5·22	−16·8	3
	5	10 38 42	−1 31·43	− 1 12·6	15	4 37 24·84	9·115*n*	75 32 30·1	0·813	+5·22	−16·6	1
	8	10 18 7	+6 6·81	+ 2 52·1	7	4 34 8·64	9·153*n*	75 25 2·2	0·813	+5·24	−17·6	4
	8	10 18 7	−3 2·08	+ 2 17·5	7	4 34 8·79	9·153*n*	75 25 2·8	0·813	+5·25	−16·6	3
	9	10 45 19	+5 1·23	+ 0 4·7	5	4 33 3·06	8·860*n*	75 22 14·8	0·817	+5·24	−17·6	4
	9	10 45 19	−4 7·60	− 0 30·4	5	4 33 3·28	8·860*n*	75 22 14·9	0·817	+5·26	−16·6	3
	18	9 17 7	+0 5·34	+18 10·8	10	4 24 23·35	9·225*n*	74 53 15·7	0·814	+5·32	−17·8	5
	18	9 17 7	−0 24·87	...	10	4 24 23·31	9·225*n*	...	...	+5·32	...	6
	22	10 19 55	+0 39·91	+ 0 40·8	10	4 21 10·36	8·041	74 37 27·9	0·823	+5·32	−18·1	7

Date. 1896.	Windsor Mean Time.	Planet−Star. Δ R.A.	Δ N.P.D.	No. of Comps.	Planet's App. R.A.	Log. $p\Delta$	Planet's App. N.P.D.	Log. $p\Delta$.	Reductions to App. Place R.A.	N.P.D.	Comp. Star.
	h m s	m s	′ ″		h m s		° ′ ″		s	″	
Dec. 22	10 19 55	−3 7·68	+ 2 23·1	10	4 21 10·35	8·041	74 37 28·2	0·823	+5·34	−17·6	5
22	10 19 55	−3 37·86	+ 5 41·9	10	4 21 10·34	8·041	74 37 28·8	0·823	+5·34	−17·6	6
23	9 39 34	−3 49·27	− 1 38·3	10	4 20 28·76	8·836*n*	74 33 26·8	0·822	+5·34	−17·6	5
23	9 39 34	−4 19·40	+ 1 40·0	10	4 20 28·81	8·836*n*	74 33 26·9	0·822	+5·35	−17·6	6
23	10 29 13	−0 3·18	− 3 29·6	10	4 20 27·28	8·624	74 33 17·6	0·823	+5·33	−18·0	7
24	9 31 45	−0 42·29	− 7 34·0	15	4 19 48·17	8·879*n*	74 29 13·2	0·823	+5·33	−18·0	7

Mean Places of the Stars of Comparison for 1896·0.

Star.	R.A.	N.P.D.	Authorities.
	h m s	° ′ ″	
1	4 38 51·05	75 33 59·3	Glasgow 1870, 1149; Madras 1882, 159; Yarnall$_3$ 2065.
2	4 40 40·10	75 39 30·3	Equatorial Comps. with No. 1.
3	4 37 5·62	75 23 1·9	Glasgow 1870, 1141; Madras 1882, 157; Glasgow 1890, 409.
4	4 27 56·59	75 22 27·7	Radcliffe 1860, 519; Brussels 1865, 1749; Glasgow 1870, 1099; Greenwich 1880, 727.
5	4 24 12·69	74 35 22·7	Brussels 1865, 1728; Glasgow 1870, 1083; Greenwich 1880, 717; Glasgow 1890, 389.
6	4 24 42·86	74 32 4·5	Brussels 1865, 1731; Greenwich 1872, 423; Greenwich 1880, 719; Glasgow 1890, 391; Cincinnati 1890, 491.
7	4 20 25·13	74 37 5·2	Radcliffe 1845, 1230; Brussels 1865, 1703; Greenwich 1872, 415.

Windsor, N. S. Wales:
1897 *February* 18.

Observations of Comets 1896 *III.* (*Swift*) *and* 1896 *IV.* (*Sperra*), *at the Radcliffe Observatory, Oxford.*

(*Communicated by E. J. Stone, Esq., M.A., F.R.S., Radcliffe Observer.*)

The following comet observations were made with the 10-inch Barclay Equatorial, using the ring micrometer, with power 100.

Comet Swift.

Date.	G.M.T.	Local Sidereal Time.	Observer.	Comet *minus* Star (corrected for Refraction only). R.A.	N.P.D.	No. of Comps.	Apparent R.A. of Comet.	Parallax in R.A. p.	Log. $(p \times \Delta)$.	Apparent N.P.D. of Comet.	Parallax in N.P.D. q.	Log. $(q \times \Delta)$.	Ref.
1896.	h m s	h m s		m s	′ ″		h m s	s		° ′ ″	″		
Apr. 20	8 57 45	10 50 28	R.	−0 16·65	−1 43·0	9	3 34 39·99	+0·68	9·593	61 46 1·8	12·1	0·840	(*a*)
28	8 44 15	11 8 28	R.	+1 29·05	+0 30·0	3	3 10 50·00	+0·79	9·6762	42 6 35·0	11·0	0·8211	(*b*)
29	9 43 35	12 11 54	C.	+1 15·50	−6 46·5	8	3 6 4·86	+0·64	9·5923	40 0 54·0	12·2	0·8739	(*c*)
May 1	10 40 18	13 16 39	W.	−3 52·73	+0 50·5	3	2 55 52·93	+0·41	9·4130	36 18 2·4	12·8	0·9082	(*d*)
1	10 40 18	13 16 39	W.	−3 58·98	+0 45·5	3	2 55 52·96	+0·41	9·4130	36 18 4·9	12·8	0·9082	(*e*)
2	10 5 49	12 46 1	R.	−1 14·39	−4 17·9	9	2 50 33·05	+0·52	9·5205	34 41 44·6	12·1	0·8912	(*f*)
4	10 44 48	13 33 0	R.	−1 50·45	+6 54·4	6	2 38 43·74	+0·29	9·2934	31 43 0·5	12·2	0·9084	(*g*)
9	10 11 45	13 19 34	R.	+0 17·80	+0 23·8	11	2 6 13·31	+0·23	9·2238	26 8 25·9	10·8	0·8952	(*h*)
11	10 37 23	13 53 9	R.	+2 53·80	−0 46·0	8	1 52 7·50	0·00	...	24 27 54·6	10·4	0·8946	(*i*)
12	9 42 52	13 2 26	R.	+0 30·88	−0 51·1	6	1 45 16·81	+0·22	9·2255	23 45 49·9	10·0	0·8868	(*j*)
14	10 53 1	14 20 40	W.	−4 3·35	−1 50·7	5	1 30 33·04	−0·26	9·3167	22 27 8·0	9·5	0·8796	(*k*)
15	11 8 46	14 40 24	W.	−3 21·89	+2 0·7	6	1 23 15·10	−0·40	9·5096	21 53 9·2	9·1	0·8675	(*l*)
29	11 32 51	15 59 45	W.	+1 34·39	−0 1·8	7	23 43 30·74	−1·10	0·0337	17 41 7·3	4·5	0·6415	(*m*)
June 15	11	...	R.	...	...	...	21 52	...	...	17 40	...	...	(*n*)

Comet Sperra.

Date.	G.M.T.	Local Sidereal Time.	Observer.	Comet *minus* Star (corrected for Refraction only). R.A.	N.P.D.	No. of Comps.	Apparent R.A. of Comet.	Parallax in R.A. p.	Log. $(p \times \Delta)$.	Apparent N.P.D. of Comet.	Parallax in N.P.D. q.	Log $(q \times \Delta)$.	Ref.
1896.	h m s	h m s		m s	′ ″		h m s	s		° ′ ″	″		
Sept. 9	10 8 2	21 20 47	W.	−0 41·99	+4 59·4	3	14 9 52·48	+0·36	9·7832	34 57 (42·)	3·2	0·7261	(*o*)
15	11 54 2	23 30 44	R.	−0 11·40	+1 19·5	13	14 51 44·46	+0·29	9·6749	36 24 3·5	4·2	0·8398	(*p*)

Observers' Remarks.

(*a*) Coma nearly round, 2′ in diameter with a condensation. Very low altitude; bright moonlight and some haze.
(*b*) Coma nearly circular; can be traced over a diameter of 3′ Nucleus diffused, about 10″ or 15″ diameter.
(*d*), (*e*), (*f*) Part of the object-glass covered by roof during these observations.
(*g*) Sky hazy. At the close of the observations nearly half of the object-glass was cut off by the roof.
(*h*) Coma 3′. Condensation small. Sky very clear. (*i*) Coma 3′. Condensation generally diffused.
(*k*) Comet is a large, diffused mass. Observations are not very satisfactory. (*l*) A momentary glimpse of a nucleus obtained.
(*m*) Comet a hazy, faint patch, with apparently a tail projecting at an angle of 300° from the north point. Difficult observations.
(*n*) Comet only very occasionally discernible. No observation could be secured; nothing but the roughest estimate possible.
(*o*) The sky has been too thick for anything more than rough work. The comet is a diffused nebulous mass, though I once suspected a nucleus. (*p*) Comet is a faint haze of about 2′ diameter, with a feeble condensation, occasionally invisible.

Assumed Places of the Comparison Stars.

Ref.	Mean R.A.	Reduction to Apparent R.A.	Mean N P.D.	Reduction to Apparent N.P.D.	Authority.
	h m s	s	° ′ ″	″	
(*a*)	3 34 56·54	+0·10	61 47 53·7	−9·0	Romberg 830
(*b*)	3 9 21·32	−0·37	42 6 14·5	−9·6	Mean of Bonn. A.G. 2730 and Greenwich Observations, 1891, 1893.
(*c*)	3 4 49·82	−0·46	40 7 49·8	−9·3	Bonn. A.G. 2667.
(*d*)	2 59 46·29	−0·63	36 17 20·9	−9·0	Cambridge (Harvard) A.G. 1379.
(*e*)	2 59 52·57	−0·63	36 17 28·4	−9·0	Cambridge (Harvard) A.G. 1381.
(*f*)	2 51 48·17	−0·73	34 46 10·9	−8·4	Radcliffe Transit-Circle Observations, 1896, Dec. 11, 15.
(*g*)	2 40 35·14	−0·95	31 36 13·6	−7·5	Helsingfors-Gotha A.G. 2515.
(*h*)	2 5 56·95	−1·44	26 8 6·2	−4·1	Radcliffe Transit-Circle Observation, 1896, Nov. 6.
(*i*)	1 49 15·31	−1·61	24 28 43·0	−2·4	Barclay Equatorial comparisons with Christiania A.G. 330 and 340.
(*j*)	1 44 47·60	−1·67	23 46 42·9	−1·9	Radcliffe Transit-Circle Observation, 1896, Nov. 6.
(*k*)	1 34 38·13	−1·74	22 28 59·3	−0·7	*Berliner Jahrbuch*, 1896.
(*l*)	1 26 38·81	−1·82	21 51 8·5	+0·1	Christiania A.G. 292.
(*m*)	23 41 57·47	−1·12	17 41 0·4	+8·7	Radcliffe Transit-Circle Observation, 1896, Oct. 31, Nov. 4.
(*o*)	14 10 33·52	+0·95	34 52 38·8	+3·8	Helsingfors-Gotha A.G. 7884.
(*p*)	14 51 55·02	+0·84	36 22 41·6	+2·4	Barclay Equatorial comparisons with Cambridge (Harvard) A.G. 4615, 4636.

Observers : W., Mr. W. Wickham ; R., Mr. W. H. Robinson ; C., Mr. E. E. McClellan.

In the computation of the parallaxes the adopted value of the Sun's mean horizontal parallax is 8″·85 ; and the geocentric distances, Δ, are taken from the *Astronomische Nachrichten*, No. 3349 (*Comet Swift*), and No. 3378 (*Comet Sperra*). For April 20 (*Comet Swift*), the value of log Δ has been inferred from that of April 22.

Radcliffe Observatory, Oxford : 1897 *April* 8.

Erratum in "Nautical Almanac," 1897.

(*Communicated by the Superintendent of the "Nautical Almanac."*)

Nautical Almanac 1897, page 464.

Reappearance of σ *Sagittarii* on August 9. The corrected numbers are :—

Sidereal time.	Mean time.	Angle from vertex.
h m	h m	°
15 47	6 33	220.

Errata in "Monthly Notices."

Vol. lvi. page 168, line 20, *for* Mean 40·013
read „ 40·133

Vol. lvii. page 380, equation (4), *for* $y = \frac{1}{(x^2+s^2)^{\frac{3}{2}}} = \frac{m}{\lambda^3}$

read $y = \frac{1}{(x^2+s^2)^{\frac{3}{2}}} = \frac{1}{\lambda^3}$.